July 2019

Challenges of Indexation in S&P 500 Index Volatility Investment Strategies

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Abstract: S&P 500 Index option-based volatility indexes have untenable risk-return profiles. These volatility indexes are not designed with consideration of important real-world risk characteristics of options and fail to represent *volatility* as a differentiated *asset-class* with relevance to the long-term utility of investors. Implications of the S&P 500 Index return distribution on the profit and loss (P&L) distribution of a directionally hedged option position are presented. The ensuing *five* cardinal characteristics of options on S&P 500 Index, central to designing viable *volatility* investment strategies, are enumerated.

Keywords: *irreducible risks, asymmetry, fat-tails, term-dependence, strike-dependence, timing, options, volatility, fiduciary, risk-management*

JEL Classification: D83, G10, G11, G12, G32

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1. INTRODUCTION

When an investment style or category is considered an *asset-class,* with relevance to the long-term utility of investors, there are indexes representing that style or category. The index behavior can provide a shorthand characterization of the *asset class's* risk-return. This characterization does not imply immaculate forecasts, rather it serves as a basis for making prudential decisions on the inclusion of that *asset-class* in a diversified portfolio consistent with an investor's risk tolerance. The market builds investable index trackers that create an entry point for obtaining exposure to the *asset-class*.

Several well-known S&P 500 Index option-based volatility indexes are shown to have untenable characteristics— uncontrolled risks and carry-costs. These indexes have failed to serve as the harbinger of *volatility* as an *asset-class*. They are not worthy of a fiduciary's consideration as an allocation to an investor's portfolio. In fact, these indexes seem to have been constructed without consideration of the long-term utility of real investors. This work presents real-world features of options on S&P 500 Index that must be accounted for to construct viable volatility investment strategies.

1.1 List of Indexes Referenced

Indexes that will be referenced in this work are enumerated here by using their ticker names on Bloomberg.

SPX Index (S&P 500 Index): A gauge of large-cap US equities using market capitalization weighted share prices.

SPTR Index (S&P 500 Total Return Index): SPTR Index reflects effects of reinvested dividends on the SPX Index.

VIX Index (Volatility Index): Purports to estimate the volatility driving the price of 30 calendar day 500 Index options (puts and calls), based on their mid-prices. This assumes that option prices reflect an *expectation* of volatility within a risk-neutral framework – i.e., does not entertain a framework where option prices reflect risk-premiums in addition to expected hedging costs.

SPVIXSTR Index (S&P 500 VIX Short-Term Futures Index Total Return): Represents long exposure to one-month maturity VIX futures contracts.

SPVXSPIT Index (S&P 500 VIX Short-Term Futures Inverse Index Total Return): Represents short exposure to one-month maturity VIX futures contracts.

CNDR Index (S&P 500 Iron Condor Index): Represents exposure to one-month put and call spreads symmetrically specified in terms of option sensitivity and carried to expiry. T-Bills are used to collateralize and to accrue interest.

PUT Index (S&P 500 PutWrite Index): Represents selling 1-month at-the-money puts and carrying them to expiry, fully collateralized with T-Bills, that are also used to accrue interest.

LBUSTRUU Index (Barclays US Aggregate Bond Index): Measures the investment grade, US dollar-denominated, fixed-rate taxable bond market (including Treasuries), government-related and corporate securities, MBS (agency fixed-rate and hybrid ARM pass-throughs), ABS, and CMBS.

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1.2 Note on Investability

The differences in investabilty of the indexes analyzed here should be recognized. The SPX Index, SPTR Index and the LBUSTRUU Index represent a basket of purchased stocks or bonds which are directly accessible to investors. These indexes provide benchmark metrics to compare with the volatility indexes.

The CNDR Index and the PUT Index are investable as their recipe defines specific option securities to be held. These indexes hold exchange-traded S&P 500 Index options, therefore their mark-to-market does not require a model or a valuation committee.

The VIX Index is calculated from S&P 500 Index option prices[1] and is not directly accessible – i.e., it is not remotely practically possible to hold a set of securities to capture its movements. Futures contracts on VIX Index, that settle based on the calculated VIX index, make SPVIXSTR and SPVXSPIT investable – with the caveat that the underlying futures settle to VIX values that are calculated and are not directly tradeable or investable.

The SPVIXSTR Index, SPVXSPIT Index, CNDR Index, and PUT Index are evaluated and assessed for their ability to capture the realities of their underlying derivatives. The return statistics of the volatility indexes examined in this work are based on a *daily* time series over the past economic cycle (*3/31/2008-3/29/2019*).

1.3 Risk-Return of Large Cap

S&P 500 Total Return Index (**Figure 1**) risk-return is displayed in **Table 1**. The compounded annual growth rate (CAGR), the volatility (upside and downside) and the Information Ratio (IR) are used throughout to assess the returns, the risk, and the risk-return profile of indexes and associated strategies. These measures are explicitly defined in the **Glossary**.

CAGR	9.5%
Volatility	19.9%
Information Ratio	0.48
Upside Volatility	18.6%
Downside Volatility	21.2%

Table 1. *SPTR Index statistics (3/31/2008-3/29/2019).*

[1]The VIX Index calculation is specified with the premise of capturing market's *expectation* of volatility in the future. As such, it is not clear that options simply price future *expectation* of volatility – for there are irreducible risks to them and it is widely believed that those irreducible risks are the rationale for a risk premium for options if there is an excess demand for them. The idealized mindset behind the VIX definition assumes a theoretical world where the returns have no jumps or serial dependence between jumps and changes in return magnitude and allows for perfectly hedging options (i.e., replication with zero residual risks); this is at the root of the untenable behavior of VIX-based Indexes.

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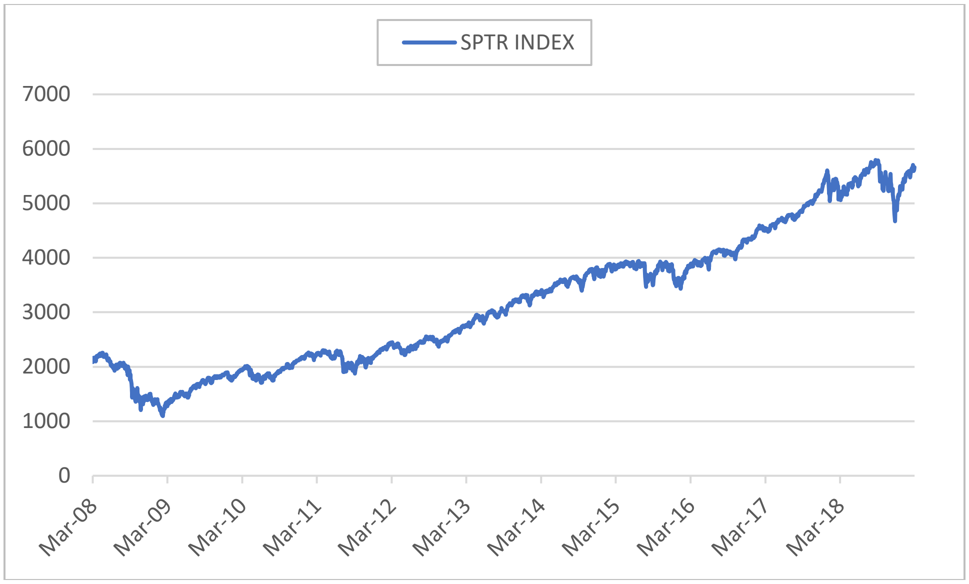

Figure 1. *SPTR Index Level.*

As a practical matter it is quite easy for an investor to participate in the S&P 500 Total Return, there are many ETFs that provide an investor exposure in return for a small management fee. Therefore, its performance statistics provide an important point of reference in judging the viability or attractiveness of a volatility investment opportunity. The SPTR Index exhibits a CAGR of 9.5% with a volatility that is approximately twice that—yielding an Information Ratio of 0.48 (**Table 1**)**.**

2. WHY VOLATILITY IS NOT YET AN ASSET CLASS

This paper first considers the volatility indexes based on VIX futures contracts and evaluates the risk-return profile of a buy and hold approach. Strategies that buy options without consideration of monetization have historically exhibited overwhelming negative carry. Strategies that sell options have made money for periods of time, however, they are notorious for catastrophic losses as they are unaware of the highly adverse asymmetric profit and loss (P&L) distribution they offer where the gains are capped, but the losses can be disproportionately large. These dynamics play out in the VIX-based Indexes.

2.1 Going Broke Buying Volatility

The SPVIXSTR Index shows why simply taking a long position in volatility is both unsustainable and ineffective. **Figure 2** shows the results of buying and holding long volatility exposure over the past economic cycle—a breathtaking long-term average rate of decline.

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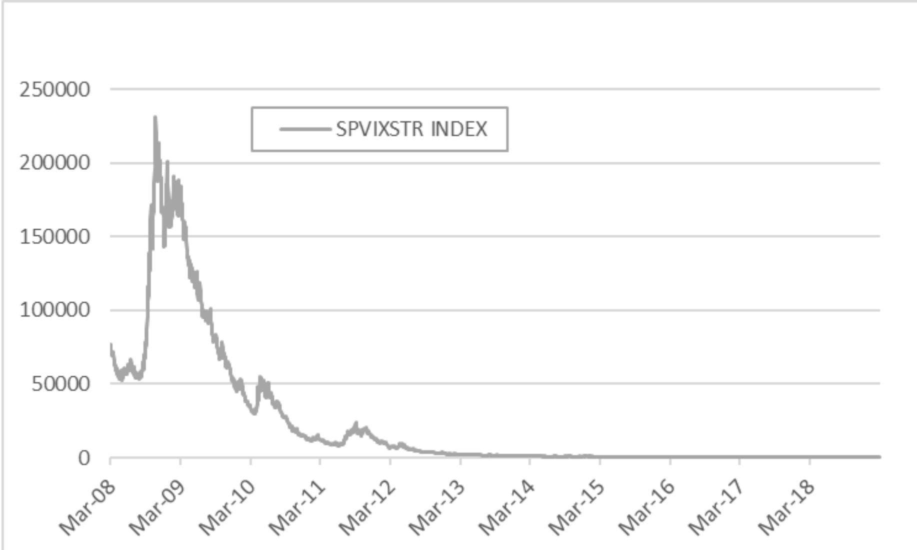

Figure 2. *SPVIXSTR Index Level.*

The overall negative drift of the SPVIXSTR Index, a long volatility exposure, is 5 to 6 times the positive drift experienced by the SPTR Index. If exposure to the SPVIXSTR Index were limited to contributing just enough to hedge an '08 like financial crisis, the portfolio would be constructed with an allocation of approximately 16% SPVIXSTR Index and 84% SPTR Index. This SPVIXSTR Index based tail-risk-hedged SPTR portfolio (**Figure 3**) goes nowhere. The long VIX futures product eats up almost all SPTR Index returns, yet, leaves significant portfolio volatility. The VIX overlaid SPTR Index Tail-Hedged Portfolio, while able to cover the 2008 downturn in the SPTR Index, only produces 1.84% CAGR with a much-worsened Information Ratio (**Table 2**).

Tail-risk mitigation in an investor's portfolio must seek to limit tail-risk without incurring overwhelming negative drift along the way. SPVIXSTR Index is a highly geared hedge that destroys the body of the SPTR Index returns. In comparison to the SPVIXSTR Index based tail-risk-hedged SPTR portfolio, a portfolio made up of just Treasuries would be a better alternative – providing higher returns and less volatility.

	SPVIXSTR Index	S&P 500 TR Index	VIX overlaid S&P 500 TR Tail-Hedged Portfolio
CAGR:	-48.7%	9.5%	1.8%
Volatility:	68.8%	19.9%	11.8%
IR:	-0.70	0.48	0.16
Upward Volatility:	79.5%	18.6%	12.8%
Downward Volatility:	58.6%	21.2%	10.8%

Table 2. *Performance characteristics of buy and hold long volatility exposures and comparison with S&P 500 Total Return Index (3/31/2008-3/29/2019).*

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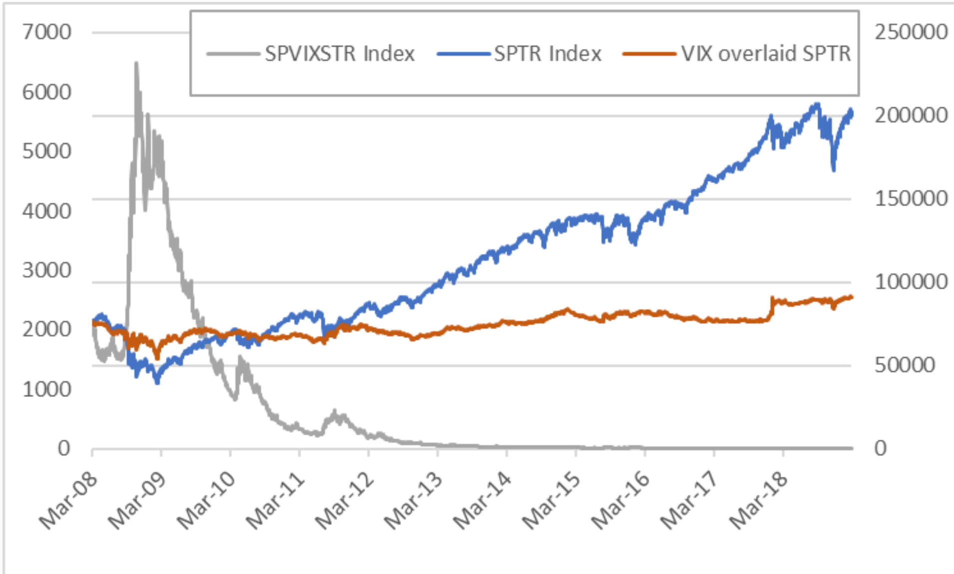

Figure 3. *SPTR Index Level, SPVIXSTR Index Level and VIX overlaid S&P Index Total Return Tail-Hedged Portfolio (constructed portfolio of 84% SPTR Index and 16% SPVIXSTR Index).*

2.2 Going Broke Selling Volatility

The SPVXSPIT Index is utilized to assess the sustainability and effectiveness of a short volatility buy-and-hold approach. It holds the same securities as SPVIXSTR Index, with the opposite direction/sign. A perpetual short volatility exposure has a distinctive long-term upward drift. However, the ride of a SPVXSPIT investor is particularly bumpy (**Figure 4**) compared to the SPTR Index (**Figure 1**). The SPVXSPIT Index has a downside volatility of 172.88% (**Table 3)** in comparison to the SPTR Index's downside volatility of 21.2% (**Table 1**). Over the time-period shown, the SPTR Index (**Table 1**) would have turned out to be a much more efficient investment when compared to the SPVXSPIT Index.

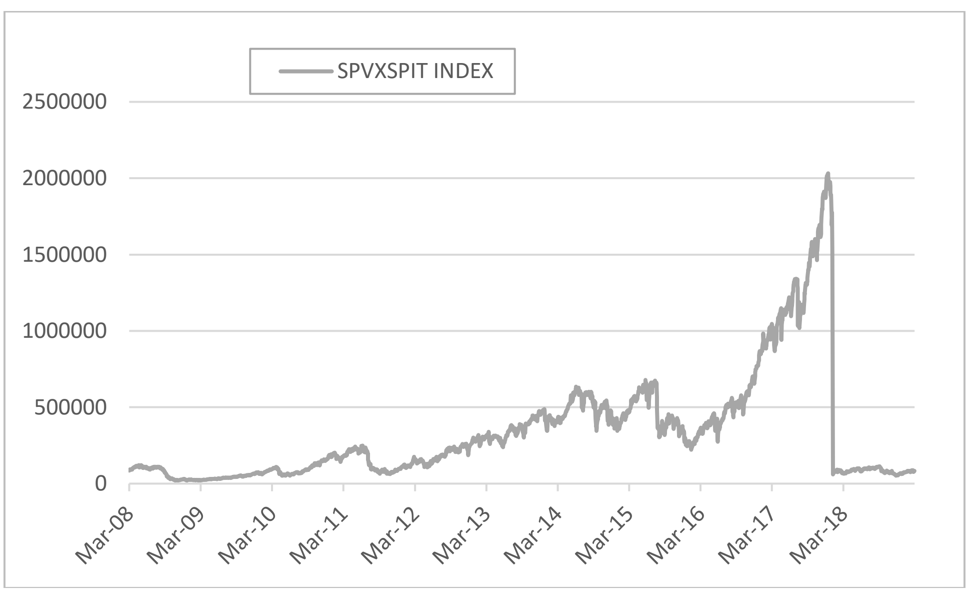

Figure 4. *SPTR Index Level.*

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CAGR	-0.2%
Volatility	119.1%
Information Ratio	0.00
Upward Volatility	55.6%
Downward Volatility	172.9%

Table 3. *Performance characteristics of buy and hold short volatility exposure from SPVXSPIT Index (3/31/2008-3/29/2019).*

On February 2nd of 2018, the short volatility index collapsed and the option-selling moniker—*weapons of mass destruction*[2]—perfectly suited the index as the SPVXSPIT Index suffered a 96% loss in one day. The potential for investment losses of this size are too large and devasting to a portfolio's long-term growth to be considered by a prudent investor. There was no economic catastrophe incurred to cause such a loss— the SPTR Index was down ~10% below its prior month peak. This mercurial reaction in VIX is not a one-time occurrence; in October of 2018, VIX moved up ~115% on a ~10% downward move in the SPX Index and again in December of 2018, VIX moved up ~119% on a ~16% downward move in the SPX Index.

Such movement of the VIX Index is intrinsic to the calculations it is based on. While the VIX Index is meant to be a measure of *fear* in the markets, it does not accurately incorporate the tails of the S&P 500 Index returns in its measurement. This is due to the fact that option prices (that VIX Index is based on) when interpreted through the Black-Scholes approach (including its follow-up risk-neutral formalisms) end up exaggerating the role of the standard deviation of S&P 500 Index returns because the theory cannot and does not consider *fat-tails* of the returns that create irreducible risks that can be orders of magnitude larger than expected option hedging cost. A large fraction of the utilized option strikes in the VIX Index fall more than one standard deviation of S&P 500 Index monthly return away from the Index level; the VIX Index incorporates strikes which by its own naïve metric cannot accurately reflect real risks. **Section 3** describes this in detail.

2.3 Going Nowhere Buying and Selling Volatility

The CNDR Index is a benchmark index which is designed to track the performance of an option trading strategy which has exposure to one-month out-of-the-money put and call spreads symmetrically specified in terms of options delta and carried to expiry. The CNDR Index has gone unimpressively nowhere over the most recent economic cycle (**Figure 5**) and exhibits similar risks of sharp downturns as seen in a perpetual short volatility exposure. The CNDR Index attempts to build a strategy which takes advantage of the positive time decay of options while managing the risks associated with options. Without reflecting an understanding of the asymmetry between buying and selling options it leaves much to be desired, with an Information Ratio of 0.04 (**Table 4**).

[2] *"In our view, however, derivatives are financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal," wrote Buffett in Berkshire Hathaway's (BRK-A, BRK-B) 2002 annual letter.*

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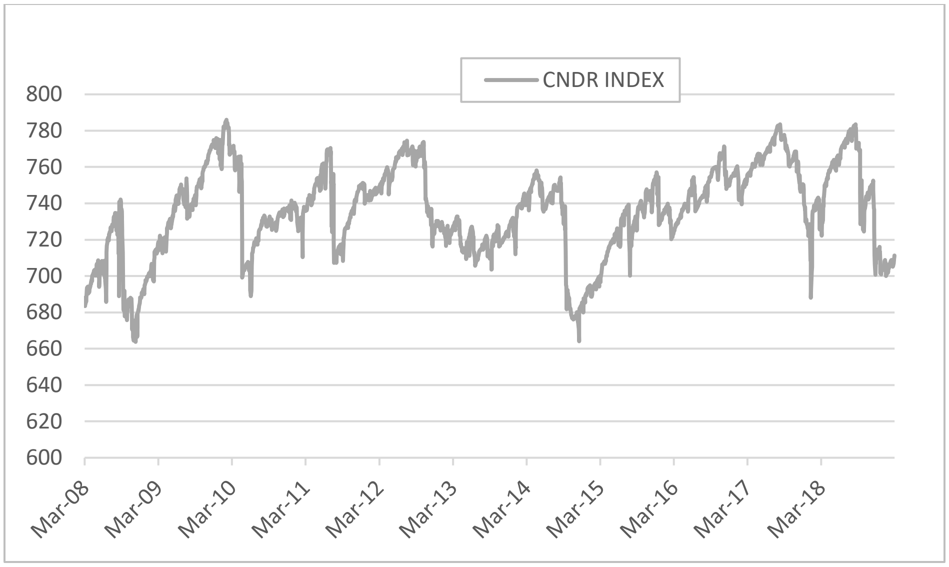

Figure 5. *CNDR Index Level.*

CAGR	0.3%
Volatility	7.6%
Information Ratio	0.04
Upward Volatility	6.0%
Downward Volatility	9.6%

Table 4. *Performance characteristics of CNDR Index (3/31/2008-3/29/2019).*

The CNDR Index does use option spreads which helps limit risks, but this alone is not enough to manage the risk inherent in selling options. The risk and harmful asymmetry of the return distribution as options approach expiry is overlooked in the Index construction as it maintains its spreads *to expiry*. An understanding of this asymmetry must be imbedded into any volatility investment strategy. This is discussed further in **Section 4.**

Additionally, the CNDR Index has an underlying assumption that risk premiums are omnipresent— that there is a positive risk premium for both puts and calls. This fallacy is discussed in **Section 4**. Risk premium is not a law or an entitlement; it is not a universal feature and can wax and wane. The CNDR Index fails to acknowledge that risk premium is not universal among puts and calls.

The thoughtlessness of this strategy in representing investor objectives is obvious when one considers the behavior of options and the supply and demand dynamics present that starkly differentiate S&P 500 Index out-of-the-money calls from puts[3]. The Index's sideways return profile and information ratio at a dismal 0.04 over the last economic cycle is a testament to the strategy's lack of understanding of the realities on hand.

[3] Due to a ready supply of out-of-the-money (OTM) call option sellers, both by retail call writing programs and by institutional investors as part of "collar" hedging programs– giving up S&P 500 Index upside by selling an OTM call to finance an OTM put purchase.

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2.4 Unimpressive Lot of Downside Protection Seller

The PUT Index is shown (**Figure 6**) over the most recent economic cycle. The PUT Index represents selling 1-month at-the-money puts that are fully collateralized with cash reserves in a money market account.

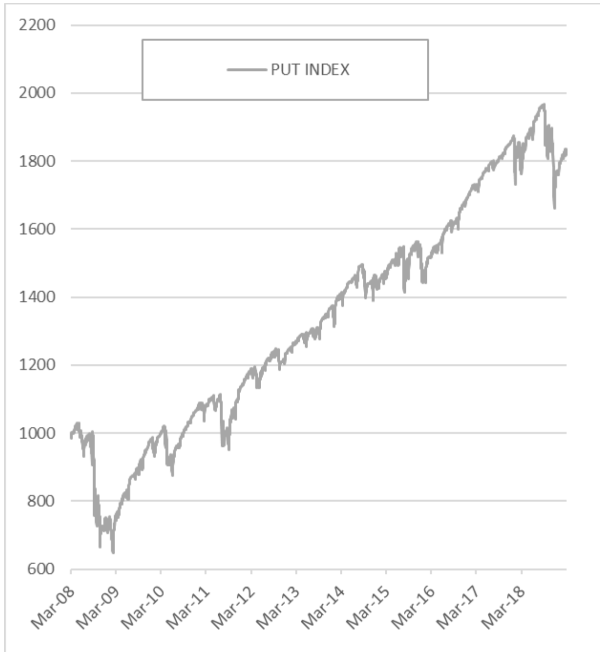

Figure 6. *Put Index Level.*

Selling a naked put with strike close to the S&P 500 Index level incurs obvious long directional exposure. This feature is different from the VIX based indexes and the CNDR Index that do not overtly take such significant directional exposure.

A portfolio is constructed of 50% SPTR Index (to mimic the initial market exposure incurred in selling the put option) with the remaining cash deployed in the LBUSTRUU Index, the aggregate bond index, over the same time period (**Figure 7**).

	PUT Index	50% SPTR + 50% LBUSTRUU
CAGR:	5.8%	7.2%
Volatility:	14.0%	9.6%
Information Ratio:	0.42	0.75

Table 5. *Performance characteristics of the Put Index and an equally weighted SPTR and LBUSTRUU portfolio (3/31/2008-3/29/2019).*

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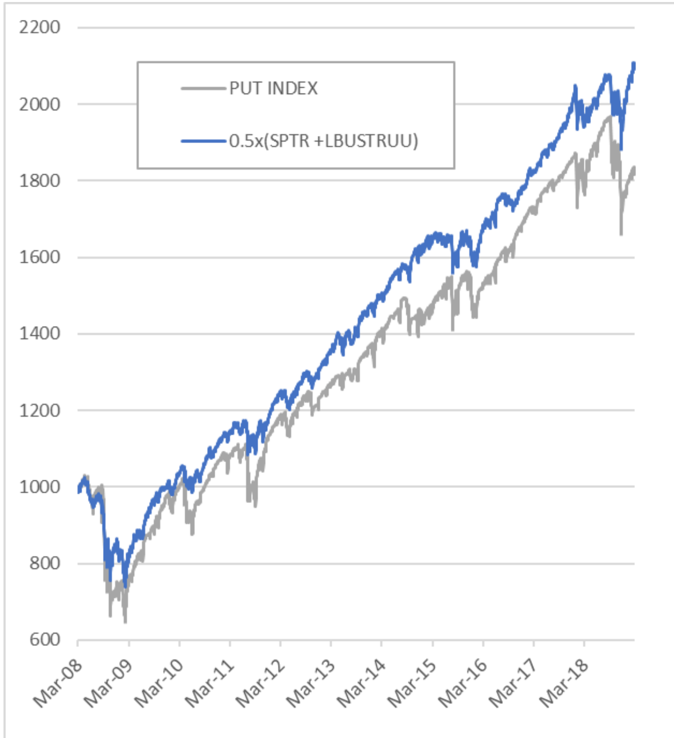

Figure 7. *Put Index Level shown with a comparison to an equally weighted SPTR and LBUSTRUU portfolio.*

The portfolio allocation with exposure to investment grade bonds and large-cap stocks shows a CAGR of 7.2% and a volatility of 9.6%— yielding an information ratio of 0.75 (**Table 5**). In comparison, the PUT Index shows a CAGR of 5.8% and a volatility of 14.0%— yielding an information ratio of 0.41. Put writing fails to produce any additional *alpha* relative to a commonplace diversified portfolio.

The nature of the put risk premium is such that just selling an at the money put and taking it to expiry is not a sustainable way to harvest the risk premium. A strategy selling insurance (optionality) has shown to be more profitable than buying insurance (optionality); however, it is too prone to negative events and does not account for the harmful nature of the asymmetric P&L distribution endemic to selling options.

Volatility indexes show untenable risk-return profiles as they were not designed with consideration of important real-world risk characteristics of options; this is analytically pursued in the upcoming **Section 3** by describing the S&P 500 Index return distribution and by a real-world approach to describe the P&L distribution of a directionally hedged option position. The ensuing cardinal characteristics of options on S&P 500 Index, central to designing viable *volatility* investment strategies, are outlined in **Section 4**.

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3. NATURE OF S&P 500 INDEX RETURNS

S&P 500 Index returns must be understood before characterizing any derivatives on the index. While an approach to treating returns as white-noise – *i.e. random noise with no discernible correlation in time and without realistic statistical patterns* – is widespread, it is not accurate nor reflective of the underlying's features. This approach may be convenient to its authors, but it is certainly not sensible for building a framework for an options-based investment strategy.

3.1 Market Rhythms and Rhymes

The market has imperfect patterns and visible clustering, shown in **Figure 8**, which arise from supply and demand dynamics and reflect cycles of fear and greed in the market.

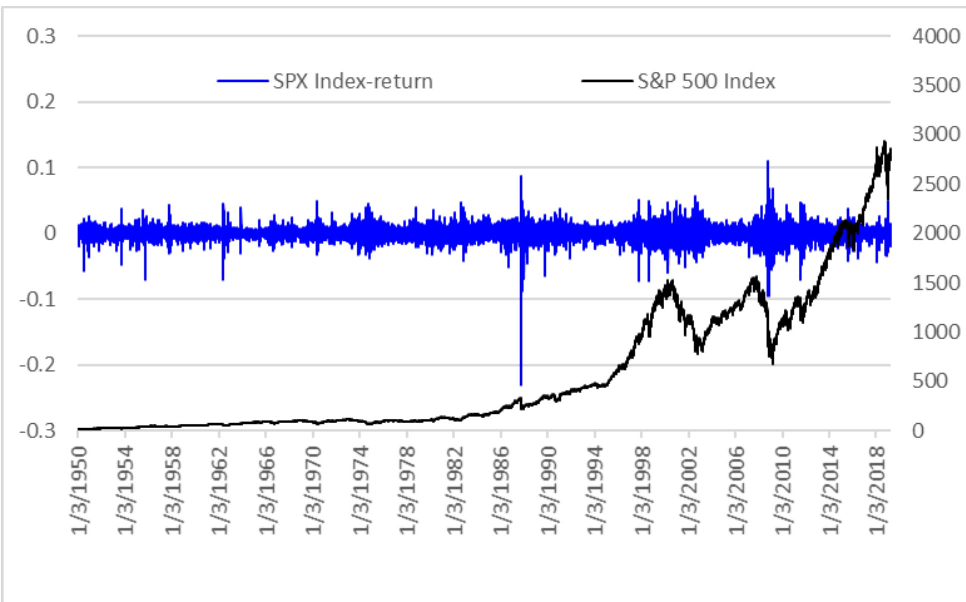

Figure 8. *The S&P 500 Index Level and S&P 500 Index returns from January 3, 1950 to March 29, 2019.*

The market's patterns—serial correlations and lead-lag tendencies—are observed in traded markets and the S&P 500 Index returns are not normally distributed as would be the case if driven from a white-noise process.

The market displays real rhythms and rhymes. The magnitude of S&P 500 daily returns ($|r|$) displays long term memory (**Figure 9**). The return sign (I) does not display slow decaying autocorrelation as seen by the return magnitude ($|r|$)– therefore neither does the return (r) itself show a high temporal correlation. However, the return sign presages return magnitude discernably. The cross-correlation between the return sign (I) and return magnitude identifies this lead-lag tendency (**Figure 10**). These signatures of temporal memory are fundamental to the return distributions, the term structure of return skewness and kurtosis, and the variability of the data set. Moreover, return magnitude is at the core of volatility which plays a central role in option economics. Significant temporal memory (**Figures 9 & 10**) also makes the case for developing metrics of attractiveness for positions sensitive to market volatility and provides an impetus and rationale for timing (more in the upcoming **Section 4**).

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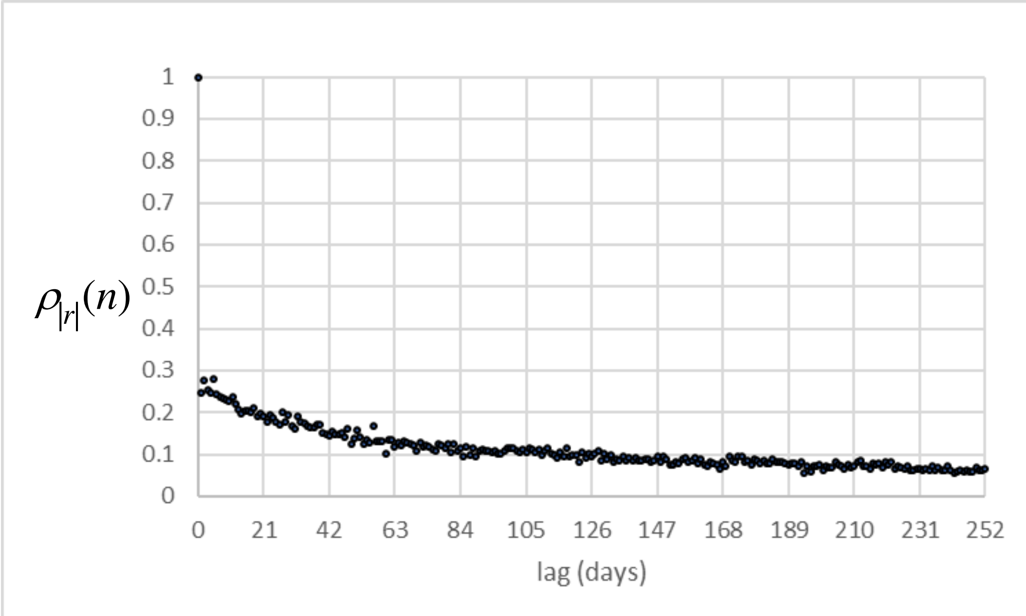

Figure 9. *The autocorrelation*[4] *of magnitude* ($|r|$) *of S&P 500 Index's daily returns (1/3/1950-3/29/2019). Wang et al* [2009] *employed this explicitly in their real-world asset-model.*

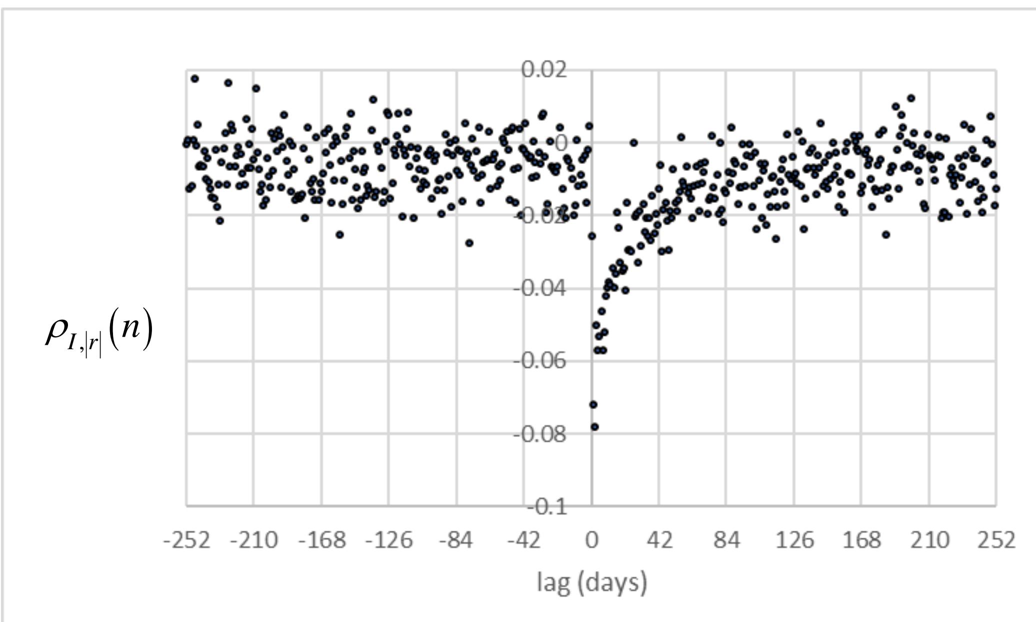

Figure 10. *The cross-correlation*[4] *of sign* (I) *and magnitude* ($|r|$) *of S&P 500 Index daily returns* (*1/3/1950-3/29/2019*). *Wang et al* [2009] *employed this explicitly in their real-world asset-model.*

[4] Defined in the Glossary.

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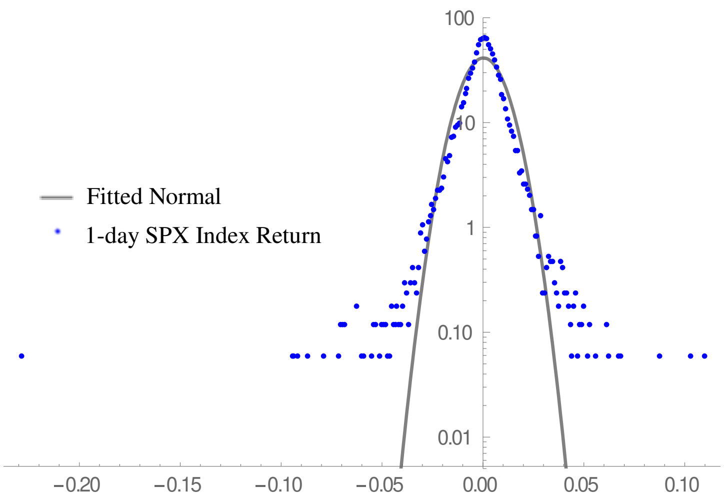
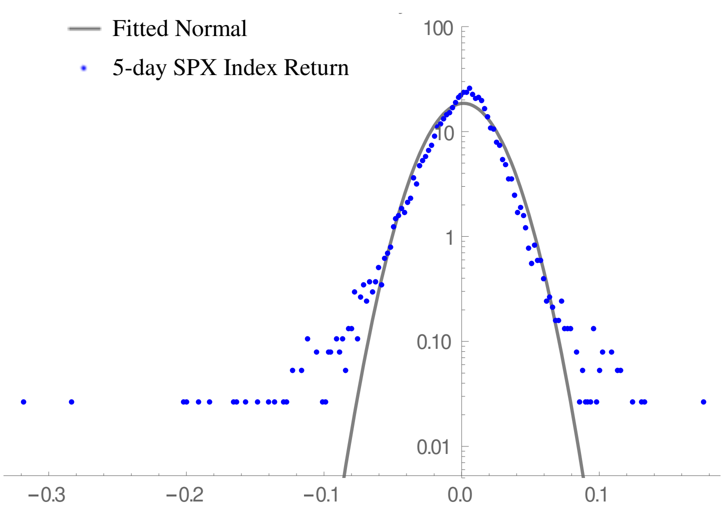
3.2 Fat-Tails and Asymmetry of Returns

A stochastic description of the market as white noise and associated Gaussian returns anticipates rare losses using the standard deviation and the associated normal distribution confidence levels. The theoretical world of white noise and associated Gaussian returns is inherently blind to extreme risks and consistently underrepresents the probability of larges losses. SPX Index return distributions are fat-tailed and non-normal. The empirical return probability distributions (1-day returns, 5-day returns, 10-day returns, 21-day returns) are shown in **Figure 11** along with corresponding fitted[5] normal distributions to emphasize the contrast of the two, especially when it comes to tail events.



Confidence Level (%)	S&P 500 One-Day Return	Return of Fitted Normal
99.9	-0.06	-0.03
99	-0.03	-0.02
95	-0.01	-0.02
90	-0.01	-0.01



Confidence Level (%)	S&P 500 Five-Day Return	Return of Fitted Normal
99.9	-0.12	-0.07
99	-0.06	-0.05
95	-0.03	-0.04
90	-0.02	-0.03

[5] Normal distribution is fitted with the same standard deviation and mean of the S&P 500 Index returns.

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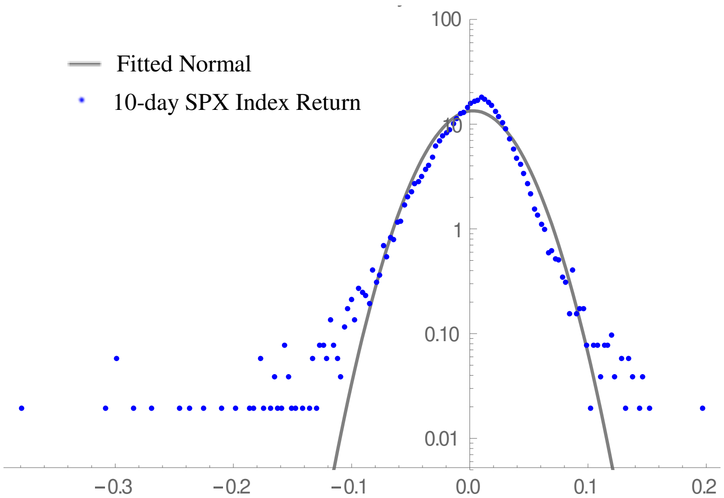
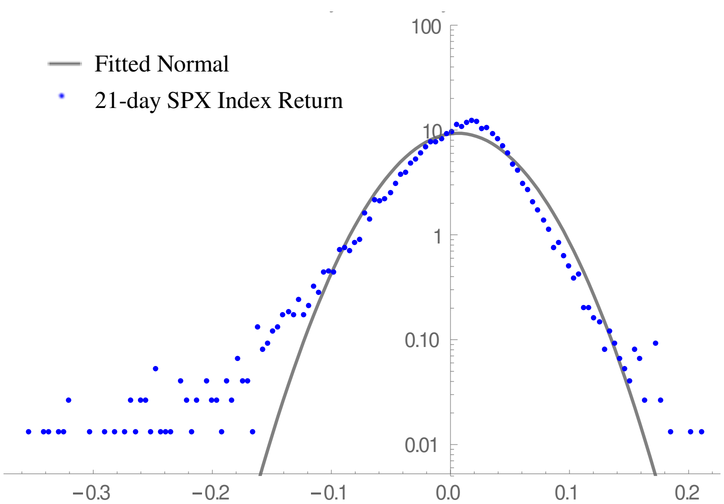


Confidence Level (%)	S&P 500 Ten-Day Return	Return of Fitted Normal
99.9	-0.18	-0.09
99	-0.08	-0.07
95	-0.05	-0.05
90	-0.03	-0.04



Confidence Level (%)	S&P 500 Twenty-One Day Return	Return of Fitted Normal
99.9	-0.26	-0.13
99	-0.12	-0.10
95	-0.07	-0.07
90	-0.04	-0.05

Figure 11. *The probability density of the 1-day, 5-day, 10-day, and 21-day S&P 500 Index Returns are shown accompanied with associated Confidence Levels. The Returns at different Confidence Levels are shown for the S&P 500 Index and for the Fitted Normal. Data shown is from 1/3/1950-3/29/2019.*

As an example, in the probability density of the real 21-day returns an investor has a 1% chance of losing 12% in comparison to the Gaussian world where an investor only has 0.25% of losing 12%. The tail-loss scenarios (99 confidence level & 99.9 confidence level in **Figure 11)** show that steep losses are much more likely than a fitted normal distribution would lead one to believe—the *six sigma* events are not as rare as some may purport.

Even more idealized than representing the market's behavior as white-noise is using this underlying assumption to characterize derivatives that have payoffs that are *non-linear* in the S&P 500 Index value. It is unwise to develop volatility investment strategies merely using information from options characterized with white-noise return.

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3.3 Implications for Options

A real-world approach is used to discern the return-risk profile of a risk taker that has sold an option contract to understand how the underlying fat-tailed, non-normal distributions factor into derivatives. This trader is avoiding taking directional exposure. If the trader has sold a SPX Index option put, she attempts to counter the option's long directional exposure by going short the SPX Index. If the trader has sold a SPX Index call option, she attempts to counter the option's short directional exposure by going long the SPX Index. This hedging is expected to cost money. In these cases, she purchases the SPX Index as it climbs and sells it when it falls, crystalizing hedging costs. The real-world approach employed in this work estimates how much money the trader is going to spend optimally hedging the option position.

The approach simulates the SPX Index with realistic *jumpiness* and *asymmetry* over the range of time scales spanning the option expiry (**Figure 12**). This Monte-Carlo stochastic description is consistent with the long-term market behavior (*unconditional*) and is also informed of more recent market outcomes (via *conditioning*) [Generalized Auto Regressive Asset Model] (Wang et al., [2009])[6]. The Monte-Carlo paths describe the non-stationarity and non-normality of the returns. Over these Monte-Carlo paths we find the risk minimizing hedging strategy (**Figure 12**). This optimal hedging strategy is not perfect. The hedging costs are not a monolith. The attempt to replicate an option is not immaculate.[7] The uncertainty and adverse asymmetry of hedging costs arises due to *jumpiness* of the SPX Index– i.e. the hedge cost probability density has a fat loss tail for a trader that has sold an option and is attempting to hedge [Optimal Hedge Monte-Carlo] (Kapoor [2010], Petrelli et al., [2010])[8]. We discern the *expected* trade economics by subtracting the average hedging cost from the option bid price; we discern the option trader's return-risk profile by dividing the *expected* economics by an estimate of uncertainty of hedging costs.

This analysis yields a hedging strategy that has an average hedge-cost and a hedge-cost distribution. The hedge-cost uncertainty is characterized in part by the standard deviation, negative standard-deviation, and tail risk measures (**Figure 12**). The resulting hedging cost probability density for an option seller-hedger has an adverse asymmetric nature which varies with strike and tenor. The asymmetry and uncertainty increase with out-of-the-moneyness and as time to expiry shrinks and can become exploding asymmetry and uncertainty in a low-volatility regime (shown and described in the **Appendix A**).

This real-world approach contrasts with measuring option prices through a normal distribution's volatility units that are mapped under immaculate hedging assumptions to a deterministic hedge-cost (without which

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[6] Exploits observed persistence and lead-lag relationships encompassing return magnitude and sign and possibly another *conditioning variable* and employs a *vector auto-regressive* framework to realistically capture the first four moments of return term structure.

[7] Bouchaud and Potters [2003] pioneered the approach adopted here to analyze options without making restrictive assumptions on the return distribution of the underlying asset. Hedge-cost uncertainty is also recognized in Derman and Taleb [2005]. Notwithstanding the recognition of irreducible hedging risks, *mainstream option quants* continue to live in a world of perfect hedging and unique hedge costs mainly for its accounting convenience, i.e., recognizing day-1 P&L on non-exchange-traded derivatives, not representing the interests of the providers of risk-capital (shareholders, investors).

[8] *Multi-Variate Variational Calculus* application where the integrals being minimized represent the residual risk (P&L Variance) of a hedging strategy and the pair of optimal functions are the hedge-ratio as a function of the S&P 500 Index and the attendant expected hedging cost. An explicit articulation of the optimal hedging strategy enables *ex-ante* assessment of residual risks.

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the Black-Scholes framework and the ensuing risk-neutral formalisms do not work). Real assets have return distributions with fat-tails. Any attempt to hedge an option on a real asset results in an uncertain P&L outcome,[9,10] characterized by a fat-tailed probability density with adverse asymmetry for the option seller-hedger. It is misguided to analyze options using a framework based on the normal distribution and the associated standard deviation and normal distribution confidence levels.

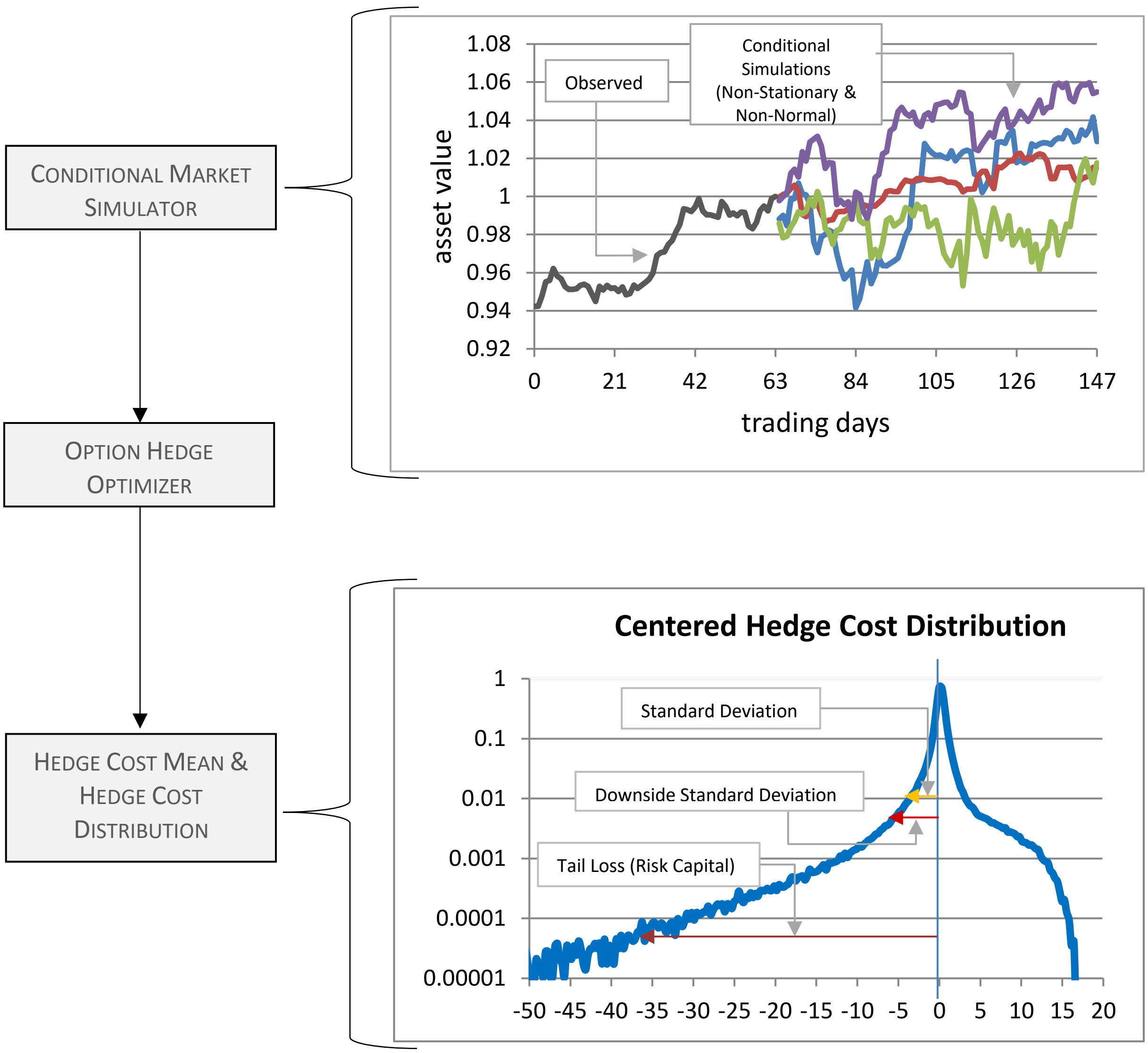

Figure 12. *Schematic of real-world approach to understanding opportunities and risks in options. The real-world asset model approach was first presented by Wang et al* [2009]. *Adopting that asset-model, the realities of the hedge-cost distribution were described in Kapoor* [2010] *and Petrelli et al* [2010].

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[9] "*Doubt is not a pleasant condition, bet certainty is absurd.*" Voltaire

[10] "*Recognizing reflexivity has been sacrificed to the vain pursuit of certainty in human affairs, most notably in economics, and yet uncertainty is the key feature of human affairs.*" George Soros

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4. FIVE CARDINAL CHARACTERISTICS OF OPTIONS

Comparison of an option *bid-price* with expected hedging costs provides an estimate of the option-seller's *expected P&L*. The *expected P&L* when compared to the *hedge-cost distribution* reveals the option's risk-return profile from a seller's perspective. This is how employing the previously described approach (**Section 3)** can discern the return-risk profile of an option *seller* and can see how the profile varies with strike and tenor, for puts and calls. Five cardinal implications for designing option strategies emerge based on this real-world risk-return assessment approach to options (Kapoor [2010], Petrelli et al, [2010]).

4.1 Disparity Between Out of the Money Puts and Calls

The differences in call and put supply-demand dynamics contribute to the characteristics and opportunity set of options (i.e. risk premium). For a sample expiry, we show the estimated return on risk capital[11] for calls and puts and see a distinct maximum for out-of-the-money (OTM) puts and distinct minimum for out-of-the-money (OTM) calls (**Figure 13**).

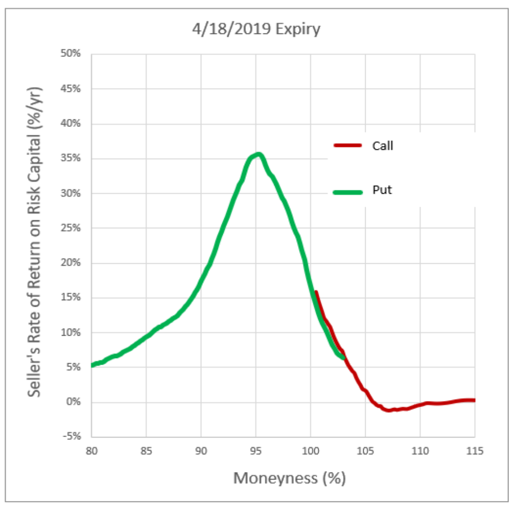

Figure 13. *A sample snapshot is shown from February 11, 2019 of the estimated risk-return profile of an option seller. The approach to discern real-world risk-return profiles of options was described in Kapoor* [2010] and *Petrelli et al* [2010]*, employing the real-world asset model of Wang et al* [2009].

The OTM call risk premium is decidedly negative (**Figure 13**), especially in comparison to the noticeably positive OTM put risk premium (**Figure 13**).

Conversations with market participants indicate that this is likely due to a ready supply of OTM call options sellers, both by institutional hedging programs as part of collar (i.e., giving up S&P 500 Index upside by selling an OTM call to finance an OTM put) and by retail call writes. Ostensibly, the existence of collar hedging programs allows portfolio allocations to equities which may otherwise be deemed too risky for certain investors and continues to have a significant imprint on option risk premium. The adverse risk premium embedded in the collar may not be in the forefront of decision making for portfolios it is overlaid on.

[11] Return on risk capital defined in Glossary.

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Recall that the CNDR Index has a prescription that is *symmetric* in calls and puts on the S&P 500 Index. The underlying assumption in CNDR Index is that risk premiums are omnipresent— i.e. there is a positive risk premium for both OTM puts and OTM calls – which is casually referred to as the *volatility risk premium.* The pointless bouncing around of the CNDR Index over the last decade (see **Figure [5]** and **Table [4]**) supports our conclusion that one must acknowledge the potential disparity between OTM put and call options when constructing a viable volatility investment strategy.

The VIX Index, by adding contributions from both calls and puts, does not seek to detect the differences between calls and puts. These distinct opportunities to buy and sell are also apparent by simply looking at the difference between fixed moneyness implied volatility (i.e., the return volatility when input to the Black-Scholes model produces a hedge cost equal to the option price with no reference to uncertainty) and subsequently realized volatility. The deviations of implied volatility from subsequent realized volatility shows the disparity between calls and puts that is obscured by the VIX Index (see **Appendix B**) and any index based on VIX futures.

4.2 Asymmetry Between Buyer and Seller

All option sell-positions and their ensuing hedging create highly adverse asymmetric P&L probability densities. The asymmetry and its adversity are unraveled by the approach described in **Section 3**. The tendency to believe options-based strategies are inherently extremely risky strategies (i.e., '*weapons of mass-destruction*'[2]) is perpetuated by option-based volatility indexes which are built without the understanding of the adverse asymmetry of returns. In the face of adverse asymmetry, surprise losses can be extremely large multiples of potential gains. An explicit addition of portfolio elements with favorable asymmetry is required to manage the adverse asymmetry of selling options.

Classical portfolio allocations (i.e., 60% SPTR Index + 40% LBUSTRUU Index) reflect the uncertainty of return and attempt to be suitable for an investor risk-tolerance. The potentially extreme adverse asymmetry of a sold option position differentiates it from classical allocations and requires special consideration.

The adverse asymmetry of an option-sell-hedge position increases with decreasing tenor and for strikes that are increasingly out-of-the-money. This asymmetry is also greater in low volatility environments relative to high volatility environments (see **Appendix B**).

4.3 Exploding Asymmetry at Expiry

The statistics of the ensuing distributions of the optimal hedging strategy for put options with constant-strike and decreasing tenor are shown from a sample model run on 31 May 2019 in **Table 6**.

The ratio of standard deviation to the expected cost of hedging—the normalized uncertainty—more than doubles when time to expiration shrinks from two-months to two-weeks. The ratio of unexpected losses (99% Confidence Level) to average hedging cost—the asymmetry—also worsens more than 2 times when time to expiry goes from two-month to two-weeks.

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	Four Months (9/30/2019 P2665)	Two Months (7/26/2019 P2665)	Two Weeks (6/14/2019 P2665)
Average Hedging Cost ($)	50.65	24.81	4.19
Standard Deviation ($)	27.48	17.20	6.62
Downside Standard Deviation ($)	35.01	22.09	9.26
99% Confidence Level Loss ($)	100.73	64.05	26.94
1% Confidence Level Gain ($)	55.48	35.76	14.63
Standard Deviation/Average Hedging Cost	0.54	0.69	1.58
Downside Standard Deviation/Average Hedging Cost	0.69	0.89	2.21
99% Confidence Level Loss/Average Hedging Cost	1.99	2.58	6.43

Table 6. *Term-dependence of residual risk for a seller-optimal-hedger of a 93% strike put.*

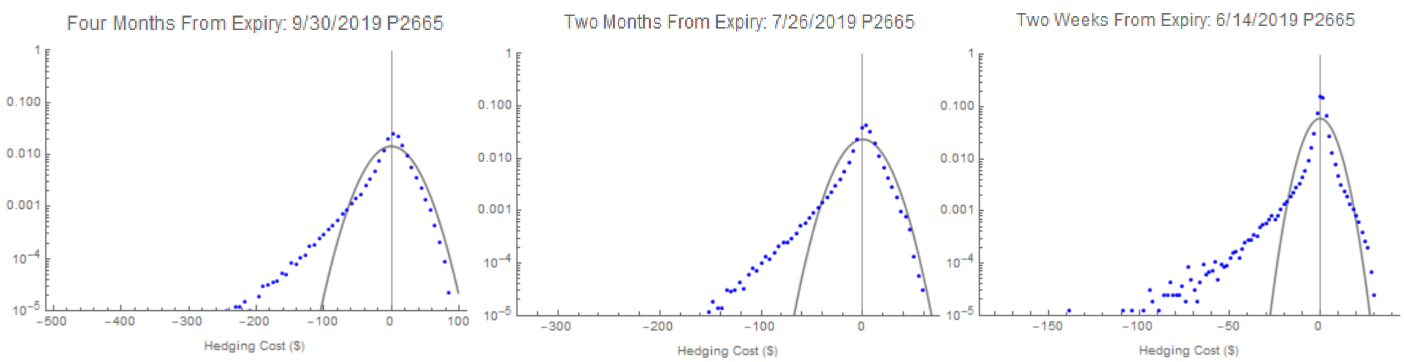

Figure 14. *Hedge cost probability distributions (centered) of seller-optimal-hedger of a 93% strike put shown with decreasing tenor (left to right). Normal distributions with the same mean and standard deviation as the hedge cost probability density are shown to provide a reference for the fat-tailed & asymmetric nature.*

Figure 14 enables a visual inspection of the probability density of the hedge cost around the mean hedging cost. The build-up of asymmetry and more heavily pronounced fat-tails for the shorter-dated expiry are apparent. An option-based strategy must be constructed with this increasing uncertainty and asymmetry in mind to construct a risk-aware strategy suitable for the investor's risk tolerance. The CNDR Index and PUT Index do not demonstrate any awareness of this as they both carry their sold options to expiry.

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4.4 Exploding Asymmetry with Out-of-the-Moneyness

The statistics of the ensuing distributions of the optimal hedging strategy for put options with constant-tenor and increasingly further out-of-the-moneyness are shown from a sample run on 31 May 2019 (**Table 7**).

	1 month 95% OTM (6/28/2019 P2745)	1 month 93% OTM (6/28/2019 P2655)	1 month 86% OTM (6/28/2019 P2475)
Average Hedging Cost ($)	29.59	10.24	0.86
Standard Deviation ($)	15.96	10.46	3.72
Downside Standard Deviation ($)	19.87	13.90	5.16
99% Confidence Level Loss ($)	54.87	41.51	6.17
1% Confidence Level Gain ($)	30.09	23.00	7.95
Standard Deviation/Average Hedging Cost	0.54	1.02	4.35
Downside Standard Deviation/Average Hedging Cost	0.67	1.36	6.03
99% Confidence Level Loss/Average Hedging Cost	1.85	4.05	7.21

Table 7. *Strike-dependence of residual risk for a seller-optimal-hedger of a one-month put.*

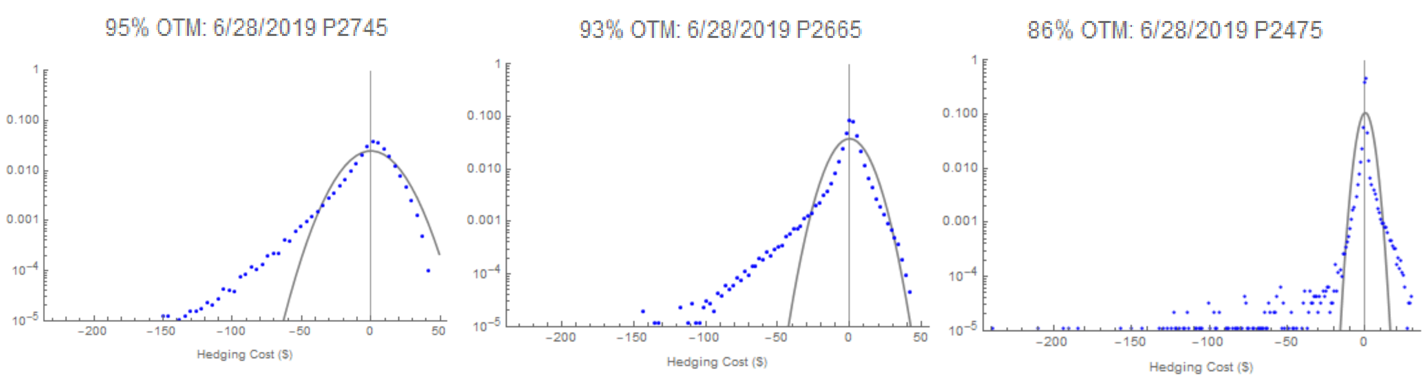

Figure 15. *Hedge cost probability distributions (centered) of seller-optimal-hedger of a 1-month expiry put shown with increasing out-of-the-moneyness (left to right). Normal distributions with the same mean and standard deviation are shown for reference to fat-tailed & asymmetric return distributions.*

The ratio of standard deviation to the expected cost of hedging– the normalized uncertainty– is 8 times greater as a one-month put gets further out-of-the-money from 95% OTM to 86% OTM. The ratio of unexpected losses (99% Confidence Level) to average hedging cost– the asymmetry– worsens more than 3 times as a one-month put gets increasingly further out-of-the-money. The increasing harmful asymmetry and exploding uncertainty is illustrated in the distributions for increasing out-of-the-moneyness at a fixed-tenor (**Figure 15**).

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As the sold put strike decreases below the SPX Index level, the expected hedging costs decrease with increasing out-of-the-moneyness. The hedge slippage metrics decrease much slower than the expected hedge costs with increasing out-of-the-moneyness. While expected volatility may explain expected hedging costs, the option price reflects the seller's recognition of hedge slippage and the need for allocating risk capital to cover that and the need for returns by the provider of risk-capital. Fitting an implied volatility to a deep out-of-the-money option price does not add any new actionable information.

An option-based strategy must be constructed with this increasing uncertainty and asymmetry with out-of-the-moneyness in mind to construct a risk-aware strategy suitable for an investor's risk-tolerance. The VIX futures-based products exhibit no awareness of this as VIX references all option strikes with quoted bid-ask prices.

4.5 Rationale for Timing

The significant temporal memory in the market's patterns (**Figure 9 and 10**) makes the case for timing of option exposure. Once metrics for the attractiveness of a volatility sensitive position (that are aware of the market rhythms and rhymes) are developed, the variation of the metrics with time and over low and high volatility regimes (**Appendix A)** can be useful for portfolio construction and management. Time-varying exposure can support the quality and sustainability of returns, in comparison to a buy and hold approach.

Timing of exposure cannot be relied on for mitigating the fundamental risks association with options. An arguably strong risk premium does not rule out disproportionate risks in any option position. The cardinal features highlighted in **Section 4.1 through 4.4** must be addressed prior to incorporating elements of timing.

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5. CONCLUSION

The untenable risk-return characteristics of the SPVIXSTR Index, SPVXSPIT Index, CNDR Index, and the lack-luster performance of the PUT Index are documented here. These volatility indexes do not reflect an understanding of options in the real-world, and therefore do not represent a prudential application of that understanding pursuant to an investor's (or risk-taker's) purpose.

The lack of a real-world understanding of options stems from the outright neglect of residual risk in the Black-Scholes model.[7] The extreme reaction of the VIX based indexes to market corrections is argued to be rooted in the well-known inability of the Normal distribution to accurately describe tail-risks of the real-world S&P 500 Index return distribution that is decisively fat-tailed.

An approach to describe the non-normal and non-stationary behavior of S&P 500 Index returns was employed to provide a Monte-Carlo simulation over which hedge optimization yielded information about the probability distribution of hedging costs. Information about the hedge cost distribution led to the articulation of cardinal characteristics of options that must be considered while designing a volatility strategy. Those cardinal characteristics are enumerated in **Table 8**, along with an assessment of whether there appeared to be any explicit consideration of them in the volatility indexes examined here.

Five Cardinal Characteristics of SPX Index Options	SPVIXTR Index	SPVXSPIT Index	CNDR Index	PUT Index
1. *Disparity Between Out of the Money Puts and Calls*	X	X	X	✔
2. *Asymmetry Between Buyer and Seller*	X	X	✔	X
3. *Exploding Asymmetry at Expiry*	✔	✔	X	X
4. *Exploding Asymmetry with Out-of-the-Moneyness*	X	X	X	X
5. *Elements of Timing*	X	X	X	X

Table 8. *The cardinal characteristics of options that are recognized by the volatility index are indicated by a* ✔ *and the properties ignored in the index construction are indicated by* a X.

None of the well-known volatility indexes receive more than two ✔. This means their construction is not aware of the cardinal characteristics of options.

None of the volatility indexes recognize that the further out-of-the-money an option is, the more adverse an option seller's asymmetry is relative to an option buyer. None of these indexes modulate their exposure with time.

The SPVIXSTR Index and SPVXSPIT Index are based on the VIX Index that references put and call options, without any cognition of the differences between them. The excessive risk baked into the SPVIXSTR Index and SPVXSPIT Index follow from the full referencing of multiple option strikes of various degree of out-of-the-moneyness with quoted bid-ask prices. The uncontrolled favorable asymmetry built into a bought option portfolio corresponding to the SPVIXSTR Index is associated with uncontrolled negative carry. The uncontrolled adverse asymmetry built into a sold option portfolio

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corresponding to the SPVXSPIT Index is associated with uncontrolled risks. Targeting a 1-month expiry, these VIX futures-based indexes do avoid referencing imminently expiring options with exploding asymmetry.

The CNDR Index limits the adverse asymmetry of the sold options by employing options spreads rather than simply selling options. A sold option spread involves buying a further OTM option for each sold option. By holding on to the spreads to expiry, however, the CNDR Index is subject to the exploding adverse asymmetry. The absence of rebalancing rules makes it susceptible to directional exposures under trending market moves and extreme adverse asymmetry under deep market reversals. The CNDR Index also fails to exploit fundamental differences between out-of-the-money calls and out-of-the-money puts.

The PUT Index recognizes the differences between put and call options on the S&P 500 Index. The adverse asymmetry of the sell option position is not addressed in that index since the options are taken to expiry and under a rapid one-way market move up the sold put can become extremely out-of-the-money, and hence subject to an extreme adverse asymmetry. The outright directional exposure taken by the PUT Index provides the pathway to a more robust replacement of it by a 50-50 allocation to SPTR Index and LBUSTRUU Index.

For volatility indexes to successfully represent volatility as an asset class worthy of a fiduciary's consideration for an investor's portfolio allocation, there must be a degree of consensus on basic risk-return features of S&P 500 Index options and there must be an attempt to intelligently reflect that in the index definitions.

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APPENDIX A: RESIDUAL RISK IN LOW AND HIGH VOLATILITY REGIMES

The hedge cost asymmetry and uncertainty becomes more pronounced with increasing out-of-the-moneyness and as time to expiry shrinks (as shown in **Sections 4.3 and 4.4**); this is even more exaggerated in a low volatility regime (see Kapoor [2010] & Petrelli et al [2010]) relative to a high volatility regime.

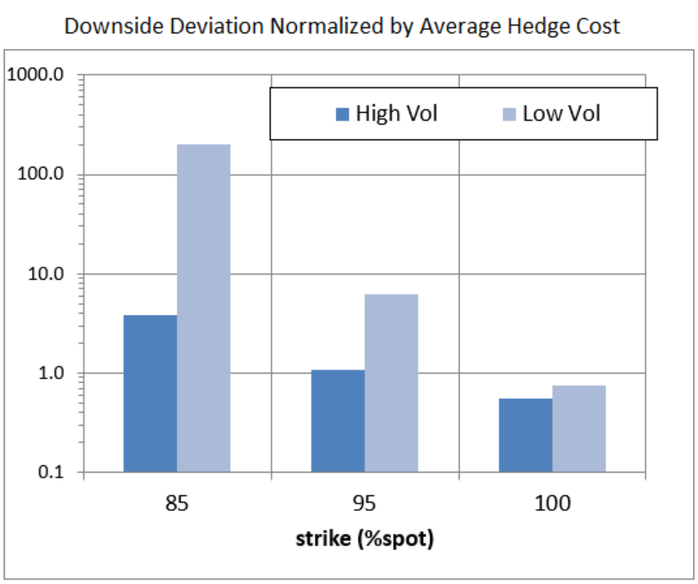
The volatility regimes shown here were differentiated based on the trailing 63-day volatility of the SPX Index. A notable deviation of realized volatility below the average realized volatility was used to pick the low volatility regime and vice-versa. The differences between these regimes is implemented through the differences in conditioning information into the market path simulator described in **Section 3** that creates an ensemble of asset value evolution paths over which hedging costs are assessed (see **Figure 12**).

Low-Volatility Regime

Strike (% spot)	**Std Deviation** (x avg hedge cost)	**Downside Std Deviation** (x avg hedge cost)	**99.9% Confidence Loss** (x avg hedge cost)
85	146.6	199.8	276.7
95	4.5	6.1	48.9
100	0.6	0.7	4.0

High-Volatility Regime

Strike (% spot)	**Std Deviation** (x avg hedge cost)	**Downside Std Deviation** (x avg hedge cost)	**99.9% Confidence Loss** (x avg hedge cost)
85	2.8	3.8	29.2
95	0.8	1.1	6.0
100	0.4	0.5	2.9



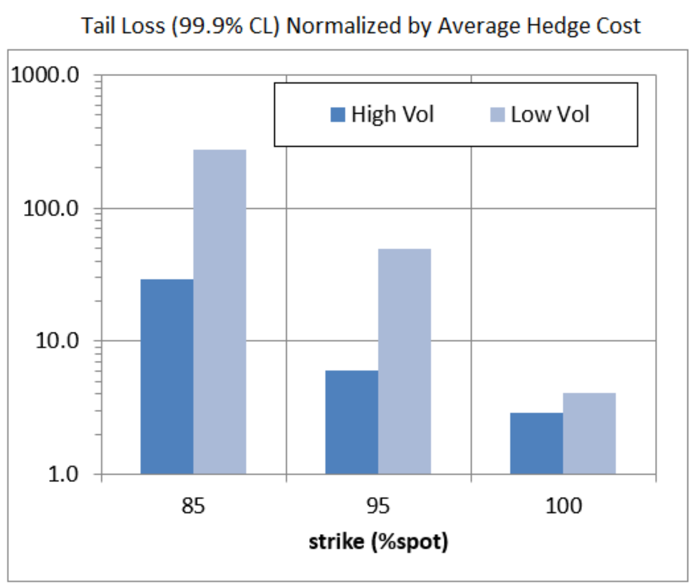


Figure 16. *Hedge cost statistics of seller-optimal-hedger of a 1-month expiry put shown as a function of out-of-the-moneyness in a low-volatility regime and a high volatility regime. The strike-dependence of uncertainty and surprise losses is visualized.*

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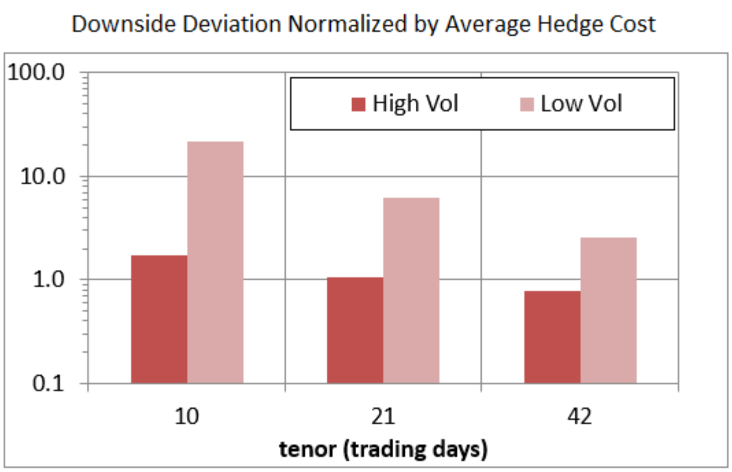
Low-Volatility Regime

Tenor (days)	Std Deviation (x avg hedge cost)	Downside Std Deviation (x avg hedge cost)	99.9% Confidence Loss (x avg hedge cost)
10	15.5	21.6	190.8
21	4.5	6.1	48.9
42	1.9	2.6	16.3

High-Volatility Regime

Tenor (days)	Std Deviation (x avg hedge cost)	Downside Std Deviation (x avg hedge cost)	99.9% Confidence Loss (x avg hedge cost)
10	1.2	1.7	10.4
21	0.8	1.1	6.0
42	0.6	0.8	4.1



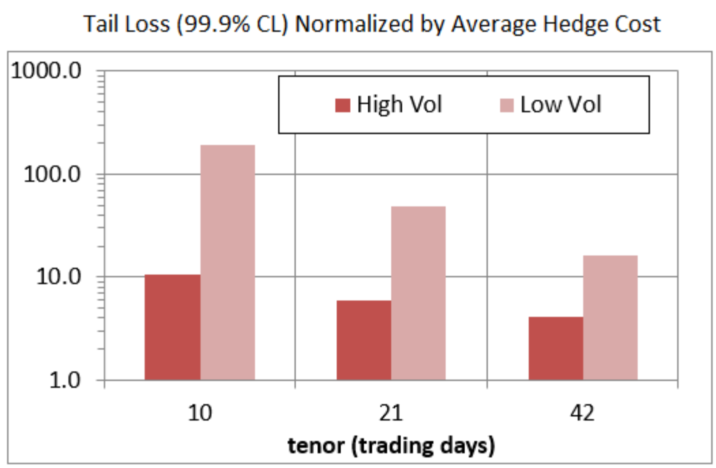


Figure 17. *Hedge cost statistics of seller-optimal-hedger of a 95% put shown with increasing tenor in a low-volatility regime and a high volatility regime. The term-dependence of uncertainty and surprise losses is visualized.*

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APPENDIX B: OUT-OF-THE-MONEY PUTS VERSUS CALLS

Deviations of the 105%/95% strike implied volatility[12] from subsequent realized volatility exhibit the differences in the opportunity elements provided by OTM call options relative to OTM put options.

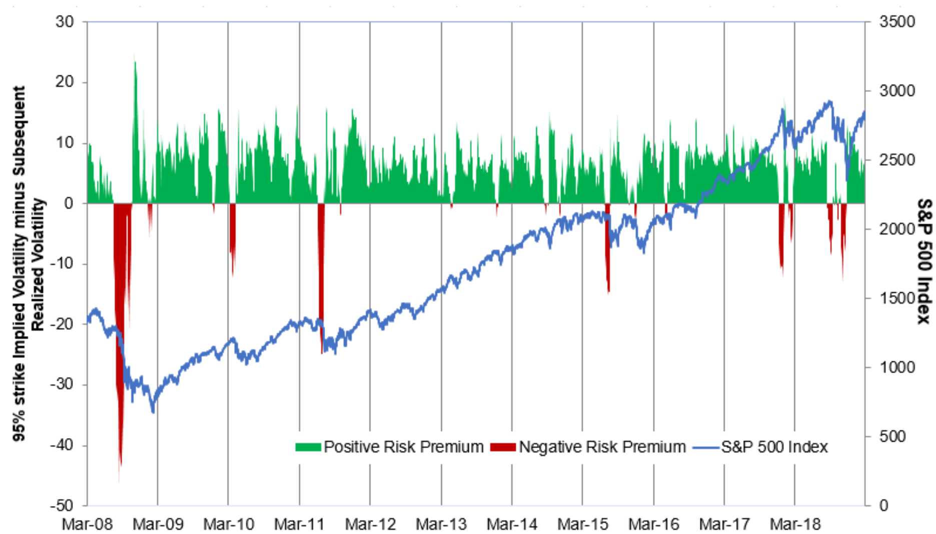

Figure 18. *OTM Put Option Risk Premium for S&P 500 (3/31/2008-3/29/2019).*

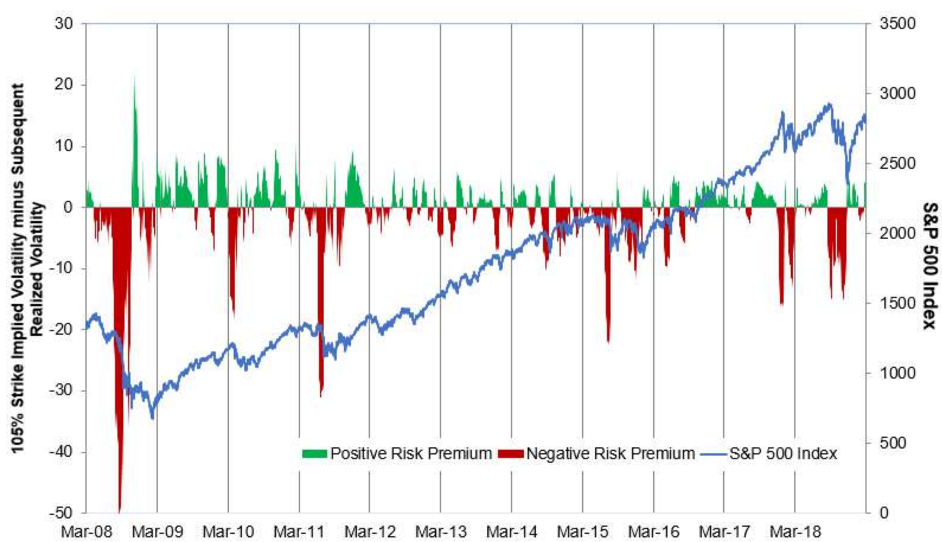

Figure 19. *OTM Call Option Risk Premium for S&P 500 (3/31/2008-3/29/2019).*

	105% Strike	95% Strike
Average Difference	-1.43	5.28

Table 9. *Average difference between implied volatility and subsequent realized volatility.*

[12] **105%/95% Strike Implied Volatility:** 30-day implied volatility at 105%/95% moneyness from the Bloomberg Listed Implied Volatility Engine (LIVE) calculated. Calculated as end of day volatilities.

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GLOSSARY

Return

Given N_a end of day values denoted by $A(t_k)$, $k \in \{0,1,2,3,......,N_a - 1\}$ we have $N_r = N_a$ -1 values of return:

$$r(t_k) = \ln\left[\frac{A(t_k)}{A(t_{k-1})}\right];\ k \in \{1,2,3,......,\ N_r\}$$

Mean Return

A central measure of returns is found by their simple arithmetic average.

$$\bar{r} = \frac{1}{N_r}\sum_{k=1}^{N_r} r(t_k)$$

Compounded Annualized Geometric Return (CAGR)

A geometric return is pertinent to describing the asset's evolution.

$$r_g = \left[\frac{A(t_{N_r})}{A(t_0)}\right]^{\frac{252}{N_r}} - 1$$

Standard Deviation of Return

The central measure of deviations of returns around their mean is described by a standard deviation

$$\sigma_r = \sqrt{\frac{1}{N_r}\sum_{k=1}^{N_r}\left(r(t_k) - \bar{r}\right)^2}$$

Annualized Standard Deviation of Return (Volatility)

If the returns were independent of each other in time, then the central measure of deviations around their mean scales with the square root of the number of periods.

$$\sigma = \sigma_r \times \sqrt{252}$$

***Annualized Upside Deviation of Returns* (*Upside Volatility*)**

With N_r^+ denoting the number of positive deviations of $r(t_k)$ around the mean, we can define a measure that quantifies the intensity of these positive deviations around the mean as follows.

$$\sigma^+ = \sqrt{252} \times \sigma_r^+ = \sqrt{252} \times \sqrt{\frac{1}{N_r^+}\sum_{k \in r(t_k) > \bar{r}}^{N_r}\left(r(t_k) - \bar{r}\right)^2}$$

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Annualized Downside Deviation of Returns (Downside Volatility)

With N_r^- denoting the number of negative deviations of $r(t_k)$ around the mean, we can define a measure that quantifies the intensity of these negative deviations around the mean as follows.

$$\sigma^- = \sqrt{252} \times \sigma_r^- = \sqrt{252} \times \sqrt{\frac{1}{N_r^-} \sum_{k \in r(t_k) \le \bar{r}}^{N_r} \left(r\left(t_k\right) - \bar{r} \right)^2}$$

Information Ratio

A measure of noise-to-signal-ratio of an investment strategy is afforded through its Information Ratio. This quantifies a rate of long-term return per unit risk.

$$\frac{\mu_g}{\sigma}$$

Autocorrelation

A measure of the correlation between a given time-series and a lagged version of itself over successive time intervals. The autocorrelation (serial-correlation) of return magnitude ($|r|$) is calculated as follows:

$$\rho_{|r|}(n) = \frac{\frac{1}{(N_r - n)} \sum_{k=1}^{N_r - n} \left(|r|(t_k) - \overline{|r|} \right)\left(|r|(t_{k+n}) - \overline{|r|} \right)}{\sigma_{|r|}^2}$$

Cross-Correlation

A measure of the correlation between two different time-series with varying time-lags—which helps identify lead and lag tendencies. The cross-correlation between the return sign indicator (I) and return magnitude ($|r|$) is calculated as follows:

$$\rho_{I,|r|}(n) = \frac{\frac{1}{(N_r - n)} \sum_{k=1}^{N_r - n} (I(t_k) - \bar{I})(|\, r \,|(t_{k+n}) - \overline{|\, r \,|})}{\sigma_I \sigma_{|r|}}$$

Sellers Rate of Expected Return on Risk Capital

$$\theta\left(T\right) = \frac{\ln\left[\Theta + 1\right]}{T} \text{(1/year)}\,;\ \Theta = \frac{\text{Bid Price - Expected Hedging Cost}}{\text{Risk-Capital}}\,;\ T\text{: duration of derivative (years)}$$

Implied Volatility

Return volatility when input to the Black-Scholes model produces a hedge cost equal to the observed option price (i.e., *fitting parameter*) with no reference to irreducible uncertainty of hedging that is shown here to be at least the same order of magnitude as the average hedge cost.

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Acknowledgements. I gratefully acknowledge discussions with the team at *Volaris Capital Management LLC* and colleagues elsewhere for feedback on drafts of this work.

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