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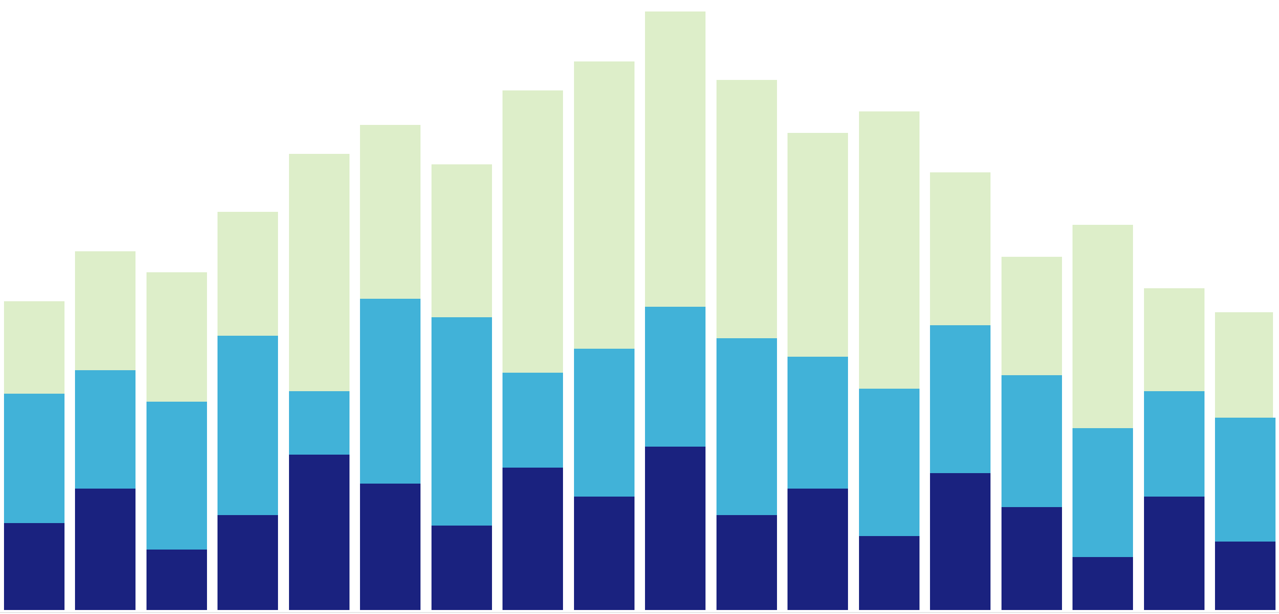

Gamma Exposure (GEX)™

Quantifying hedge rebalancing in SPX options

SqueezeMetrics
Research

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A survey of academic literature and the methods of market practitioners suggests that the relationship between the equity option market and its underlying stocks is still very poorly understood.

Historically, the published research on the effect of equity options on underlying prices has been limited to theorizing about the effects of option introduction and expiration. Meanwhile, practitioners refer almost exclusively to the volatility and variance figures implied by the quotes of only a small strip of strike prices, interpolated across two near-term contracts.

Since few have even acknowledged the pervasive impact of existing options on their underlying stocks, none have put forward a pragmatic model for predicting the day-to-day impact of those options. Gamma Exposure (GEX)™ is the first attempt at such a model.

An Illustration

The S&P 500 Index (SPX) has a large and deeply liquid market for cash-settled options. The commonly cited CBOE Volatility Index (VIX) is derived from these options. As a calculation of 30-day option-implied variance, VIX uses a selection of quoted prices to derive what can be considered, put simply, an estimate of how much the S&P 500 will move in the future.

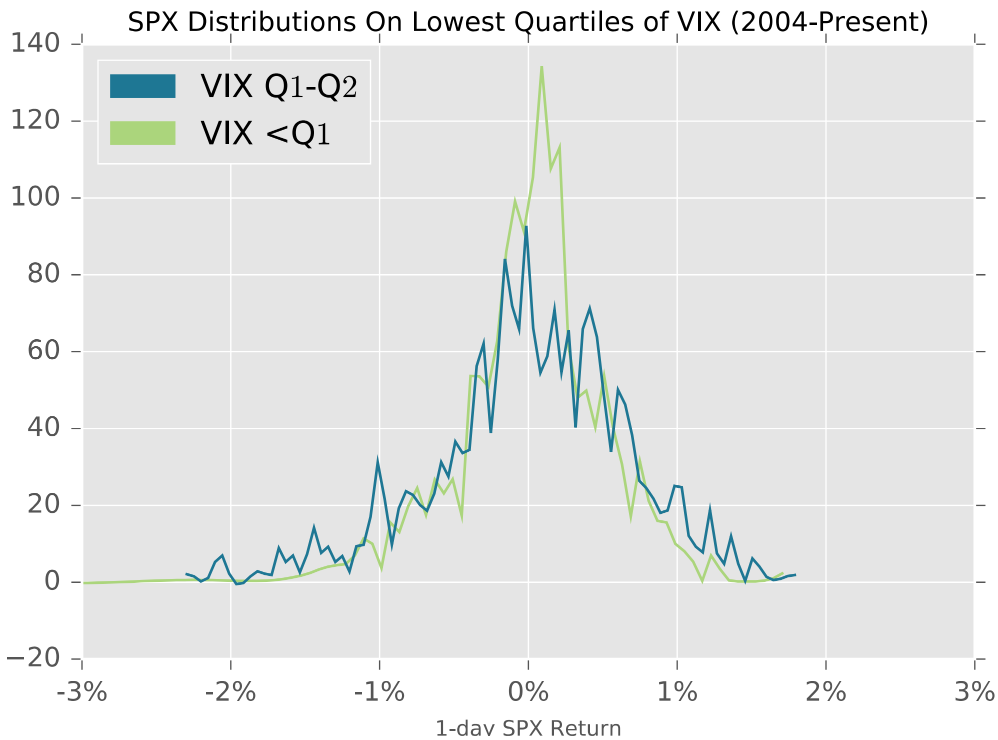
** CBOE (see References)*

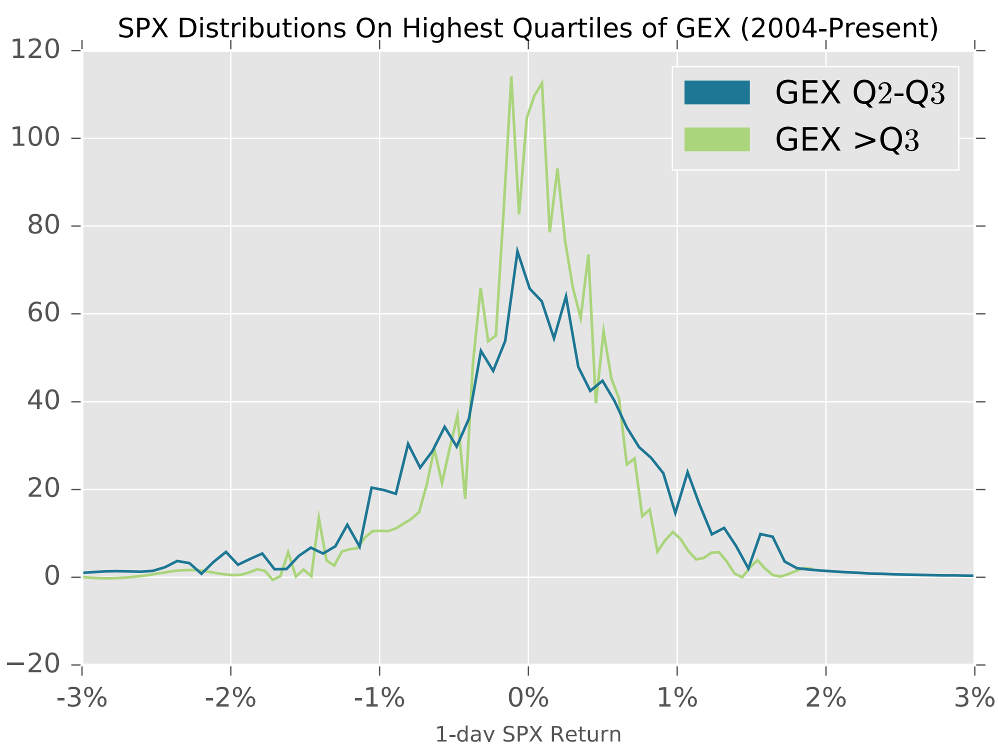
As you might expect, VIX is effective at predicting 30-day future realized volatility—the CBOE places the correlation at 0.75. But the VIX is more strongly correlated (0.85) to the realized volatility of the *previous* month.* This calls into question whether VIX can be used to forecast SPX variance.

In that vein, the VIX clearly fails to generate suitably distinguishable future stock return distributions at its lower readings. The 1-day standard deviation of SPX returns following the lowest quartile of spot VIX closes (0.51%) is not a great deal different from that of the second-lowest quartile (0.66%). A VIX of 12 means about the same as 15 when it comes to predicting variance.

*** Like low VIX, high GEX predicts low volatility.*

When instead we compare the highest and second-highest** quartiles of GEX to subsequent SPX variance, the 1-day standard deviations are 0.55% and 0.85%, respectively. On an index trading at 2000, the difference between 0.66% and 0.85% is a 4-point difference in range every day.


SPX Distributions On Lowest Quartiles of VIX (2004-Present)
VIX Q1-Q2
VIX <Q1
140
120
100
80
60
40
20
0
−20
-3%
-2%
-1%
0%
1%
2%
3%
1-day SPX Return


SPX Distributions On Highest Quartiles of GEX (2004-Present)
GEX Q2-Q3
GEX >Q3
120
100
80
60
40
20
0
−20
-3%
-2%
-1%
0%
1%
2%
3%
1-day SPX Return

We believe that the greater granularity of the GEX distributions suggests that there is some element of market volatility that is simply not able to be captured by the VIX model, or indeed any other variance metric based on quoted option prices. Rather than prices, GEX concerns itself with the quantity and characteristics of all existing option contracts at all strikes, and at all expirations—and the market participants who trade them.

Dynamic Hedging

The predictive power of GEX is essentially driven by the necessity of option dealers' (market makers') re-hedging activities. In order to limit risk and realize profit, an option market-maker must limit his exposure to deltas.

If, e.g., a market-maker sells a single, 20-delta put contract to an investor, he must then short-sell approximately 20 shares of the underlying stock* in order to (temporarily) neutralize the convexity effect of the option's gains and losses. Since the convexity itself cannot be hedged away, the market-maker must commit to re-hedging the option to its new delta whenever the underlying price changes enough to justify action.

** It is easier to speak of dynamic hedging in terms of shares, but SPX options have no underlying shares, and can be hedged with futures.*

If in one case the price of the underlying falls and the put delta rises from 20 to 50, the market-maker will be compelled to short-sell an additional 30 shares of the underlying to stay delta-neutral. If instead the underlying rises and the put delta falls to 0, the market-maker would buy back the previously shorted shares. Thus, a market-maker is essentially committed to a predictable and quantifiable regimen of buying and selling stock.

Four Assumptions

The size and liquidity of the equity option market is what makes it possible to glean predictive information from the impact of hedging activity, but it also introduces some challenges. With a few assumptions, we attempt to overcome those challenges:

1. *All traded options are facilitated by delta-hedgers.* This is to say that every option contract is either bought by, or sold by, a market participant whose business is to profitably manage a book of options.

2. *Call options are sold by investors; bought by market-makers.* It is difficult to determine the “direction” of a trade in an ultra-liquid market, as in the case of SPX options. A vast majority will trade at the midpoint of the bid and ask. It is apparent from an analysis of skew, open interest at strike, and (circularly) the effects of GEX, however, that the practice of call overwriting (and stock collaring) drives the market for calls.

3. *Put options are bought by investors; sold by market-makers.* As with calls, puts are primarily used by investors who are already exposed to the underlying market, and who are looking to modify the return profile of their portfolio by using options. The “protective put” commands a premium for

this reason, thus influencing the apparent vertical skew of index options.

4. *Market-makers hedge precisely to the option delta.* If market-makers hedged their deltas every time an option's delta changed, they would be trading incessantly. In reality, market-makers utilize “hedging bands” to balance the twin challenges of hedging costs and delta risk. Since it is not feasible to gauge the breadth of every market-maker's hedging band, we simply use the delta of the option.

With these assumptions we can compute GEX.

Computation

The gamma of an option is the first derivative of its delta, expressed as the rate of change of delta per 1-point move in the underlying. To calculate the GEX of an option, we need to determine the share impact of that potential change in delta on a market-maker's book.

Thus, if the gamma of a single 50-delta call option is 10, we can assume that a market-maker will re-hedge that option to either 40 or 60 delta in the event of any ±1 point move in the underlying. In either case, the market-maker will need to trade 10 shares. E.g., if the underlying moves up 1 point and the new delta is 60, the market-maker will short-sell 10 shares. If the underlying moves down 1 point and the new delta is 40, the market-maker will buy back 10 shares.

In the case of a put option (assumed to be held short by a market-maker), the mechanism is the same, whereby re-hedging involves selling shares as delta rises and buying as delta falls.

To calculate the GEX (in shares*) of all call options at a particular strike price in a contract:

$$\text{GEX} = \Gamma \cdot \text{OI} \cdot 100$$

** When computing for SPX, we denominate GEX in dollars.*

Where Γ is the option's gamma, OI is the open interest in the particular option strike, and 100 is the adjustment from option contracts to shares of the underlying.

In the case of put options:

$$\text{GEX} = \Gamma \cdot \text{OI} \cdot (\text{-}100)$$

The share adjustment is negative here because for market-makers, where calls represent long gamma, puts represent short gamma.

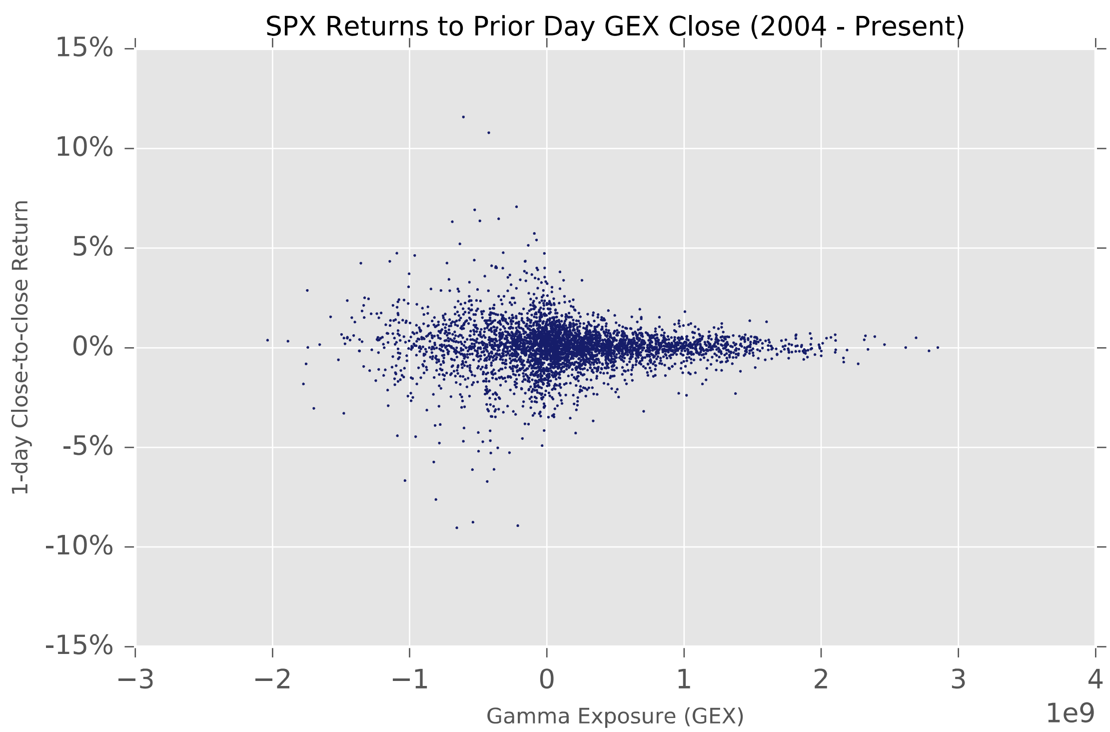
The GEX of a stock (or index) is the summation of GEX at every strike price in every available contract.

Market Impact

Using the above computations, a GEX figure that is *positive* implies that option market-makers will hedge their positions in a fashion that stifles volatility (buying into lows, selling into highs). A GEX figure that is *negative* implies the opposite (selling into lows, buying into highs), thus magnifying market volatility.

A corollary is that a GEX figure approaching zero will allow the market to move naturally and without any particular interference from market-makers' re-hedging activities.

The figure below, plotting SPX returns against SPX options' prior day Gamma Exposure, bears this out.



Note the exponential increase in volatility as GEX trends below zero, and the gradual tightening of the distribution as GEX rises.

It would be difficult to come up with an alternate explanation for this behavior.

Beyond SPX

As implied by its computation, GEX can be used to forecast return distributions in any optionable equity with enough open interest to influence prices. Aggregating GEX data across a basket of stocks or sectors in order to gauge potential risk or return potential is another plausible use-case.

An investment process that selected potential investments from the components of the S&P 500 by screening for a GEX near zero yielded excess returns.

All of the above are subjects for further investigation.

Conclusion

If investors continue to look toward the option market for alpha signals and risk assessments, they would do well to consider Gamma Exposure as a smarter alternative to price-derived volatility and variance estimates.

The key deficiency in using option prices to gauge future volatility is that no two market-makers' books are the same, and a tight spread from any one market-maker completely obscures the risk appetite of every other. This problem is readily ameliorated by computing the GEX of options known to be in circulation and deriving projected return distributions from the historical market impact of those contracts.

And so, when—in light of the evidence—investors eventually acknowledge that the option market *does* have a truly pervasive, day-to-day impact on the paths and volatilities of stock prices, we think that it is a natural next step to consider GEX an essential addition to the equity investment process.

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