Dissecting the iPath S&P 500 VIX Short-Term Futures ETN(VXX)

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Abstract

Investor demand for trading vehicles that allow one to express a view on volatility, irrespective of underlying price direction, has spawned a plethora of exchange traded products. Some have been wildly successful, while others have failed. One relative success has been the exchange traded note (ETN) referencing the S&P 500 VIX Short-Term Futures Index Total Return, or VXX. However, despite its success as a product introduction it turns out that VXX does not entirely succeed as a vehicle for trading or hedging volatility (as defined by the VIX index). Challenges for the VXX are driven mainly by persistent VIX futures curve contango, and by how the underlying reference index, SPVIXSTR, is defined, which in turn has the undesirable quality of creating non-perfectly positive correlation with the VIX Index. Taken together, these shortcomings can make VXX a poor choice for trading or hedging volatility.

1 Introduction

The iPath S&P 500 VIX Short-Term Futures ETN, or VXX, is an unsecured debt security issued by Barclays Bank PLC. It is designed to track the performance of the S&P 500 VIX Short-Term Futures Total Return Index, which offers exposure to a daily rolling long position in the first and second month VIX futures contracts and reflects the implied volatility of the S&P 500 Index at various points along the volatility forward curve. It is marketed as a vehicle for those those wishing to express an outright view on the direction of volatility, for hedging, and for volatility arbitrage. For example, if one believes equity volatility will rise, then one buys VXX. VXX can also be used to hedge an equity portfolio, since volatility tends to rise as stock prices fall. Hence a long VXX position should act as a hedge to a long equities portfolio. Further details on VXX can be found on the iPath web site `http://www.ipathetn.com/us/product/vxx/`.

This short paper will review the calculation mechanics and primary features of the VXX. In particular it will describe the calculation methodology of the SPVIXSTR index underlying the VXX net asset value (NAV). The weighting scheme applied to the front month and second month VIX futures contracts as well as the daily futures roll within a given roll period will be detailed. It will be seen that the daily futures roll component of the SPVIXSTR Index makes only a small contribution to the SPVIXSTR daily total return and can in general be ignored. However, what is more interesting is the way in which the weights evolve within a roll period, and what this implies for VXX returns performance and risk management. Lastly, the negative impacts arising from VIX futures curve contango, and VXX vs. VIX Index correlation, will be reviewed.

2 Calculation of the S&P 500 VIX Short-Term Futures Index TR Index

The S&P 500 VIX Short-Term Futures Index TR Index, or SPVIXSTR Index, models the returns from holding a VIX futures position that is rolled daily throughout the period between futures contract expiration dates. Given SPVIXSTR Index is a total return index, it incorporates the interest accrual on the return of the notional value of the index as well as reinvestment of returns and interest. Interest accrues based on the 3-month U.S. Treasury rate. Specifically, SPVIXSTR Index measures the return from a rolling long position in the first and second month VIX futures contracts. The index rolls daily from the first month contract to the second month contact. The NAV of the ETN linked to the SPVIXSTR total return, VXX, is calculated as per the following formulas:

$$IndexTR_t = IndexTR_{t-1} * (1 + CDR_t + TBR_t) \tag{1}$$

where:

$IndexTR_{t-1}$ = The index Total Return on the preceding business day, defined as any day SPVIXSTR Index is calculated.

$$CDR_t = TDWO_t/TWDI_{t-1} - 1 \tag{2}$$

where:
t-1 is the preceding business day.
$TWDO_t$ is the Total Dollar Weight Obtained on t, as determined by the following formula:

$$TWDO_t = \sum_{i=1}^{2} CRW_{i,t-1} * DCRP_{i,t} \tag{3}$$

$TWDI_{t-1}$ is the Total Dollar Weight Obtained on t-1, as determined by the following formula:

$$TWDI_{t-1} = \sum_{i=1}^{2} CRW_{i,t-1} * DCRP_{i,t-1} \tag{4}$$

where:
$CRW_{i,t}$ = Contract Roll Weight of the i^{th} VIX futures contract on date t.
$DCRP_{i,t}$ = Daily Contract Reference Price of the i^{th} VIX futures contract on date t.
TBR_t = Treasury Bill Return as determined by the following formula:

$$TBR_t = \left[\frac{1}{\left(1-\left(\frac{91}{360}\right)*TBAR_{t-1}\right)}\right]^{\frac{Delta_t}{91}} - 1 \tag{5}$$

where:
$Delta_t$ = the number of calendar days between the current and previous business days.
$TBAR_{t-1}$ = the most recent weekly high discount rate for 91-day U.S. Treasury bills effective on the preceding business day.

The roll period starts on the Tuesday prior to the monthly CBOE VIX futures settlement date (the Wednesday occurring 30 days prior to the S&P 500 option expiration for the following month). Therefore the indices are rolling on a daily basis. On the business date after the current roll period ends the following roll period will begin. When calculating the total return of the SPVIXSTR Index, the Contract Roll Weights, $CRW_{i,t}$ of each futures contract in the index, on a given business day, t, are determined as follows:

$$CRW_{1,t} = 100 * \frac{dr}{dt} \tag{6}$$

$$CRW_{2,t} = 100 * \frac{dt-dr}{dt} \tag{7}$$

where:

dt = Total number of business days in the current roll period beginning with and including the starting VIX futures settlement date and ending with, but excluding, the following VIX futures settlement date. The number of business days stays constant when a new holiday is introduced intra-month, or due to an unscheduled market closure.

dr = Total number of business days with a roll period beginning with, and including, the following business day and ending with, but excluding, the following VIX futures settlement date. The number of business days includes any new holiday introduced intra-month up to the business day preceding such holiday.

What is interesting to note regarding (6) and (7) is how they are applied within (3) and (4). By inspection it can be seen that the weights used to calculate the SPVIXSTR Index daily return via (2) are lagged by one business day. What this means is that daily returns are not influenced by changes in the weights. This is by design and thus only changes in the front and second month VIX futures prices, plus the interest earned on the daily roll, matter. Hedging or replication of a VXX position by taking weighted positions in the front and second month VIX futures contracts should take the lag into account.

At the market close on the Tuesday corresponding to the start of the new roll period, all the weight is allocated to the first month contract. Then on each subsequent business day a fraction of the first month VIX futures contract position is sold and an equal notional amount of the second VIX futures contract is bought. In this way the initial position is progressively moved to the second month contract until the following roll period starts and the second month VIX futures contract becomes the new first month VIX futures contract, and the VIX future contract maturing after that one becomes the new second month VIX futures contract.

In addition to the rolling mechanics described above, it's important to note that the weight of each index component is applied so as to ensure the SPVIXSTR Index daily return is driven only by price changes of the futures contracts and the Treasury bill return, and not from the changes in the weights themselves. This can be confirmed from equation (2).

3 VXX as a Volatility Trading Vehicle

VXX was designed as a vehicle for short to medium-term trading of volatility as expressed by the VIX index[1]. For example, if one believes the VIX Index will rise, then one could purchase shares of VXX. Unfortunately, the only way for the average investor to express a view on the VIX Index is via futures-based products[2], which have historically suffered due to the effects of futures curve contango[3]. In fact, when VXX debuted on January 29, 2009, its price was 400, but at the time of this writing stands at 16.19! The underlying index itself has cumulatively lost over 93% of its original value[4]! As can be seen in Figure 1, nearly all of this underperformance can be attributed to the persistent contango of the VIX futures curve.

Figure 1: VIX futures curve has been in contango most of the time.

It's fair to say that any futures contract-based index, and by construction any ETN referencing that index, will significantly suffer if the underlying futures curve is mainly in a state of contango[5]. In other words, contango will generally be working against anyone owning a long position in VXX.

However, what makes things worse for VXX is that the weights associated with the two futures contracts used to construct SPVIXSTR Index will also combine to magnify the underperformance relative to the VIX Index. The weights evolution for the front month and second month VIX futures contracts within a roll period can be observed in Figure 2.

[1]The Chicago Board Options Exchange Volatility Index, or the VIX, reflects a market estimate of future volatility, based on the weighted average of the implied volatilities for a wide range of strikes. 1st & 2nd month expirations are used until 8 days from expiration, after which the 2nd and 3rd month expirations are used.

[2]Apart from VXX there are other exchange traded products such as the VXZ, TVIX, and XIV, although details of these won't be covered

[3]Contango is a market condition whereby prices for a commodity or financial instrument are higher on forward futures contracts than for near-term contracts. What this means practically is that as one rolls from the first month contract to the second month contact, he is buying high and selling low, which is a guaranteed way to lose money.

[4]Which is roughly -34% per annum.

[5]The opposite is true for futures curves in backwardation. Backwardation is a market condition whereby prices for a commodity or financial instrument are lower on forward futures contracts than for near-term contracts.

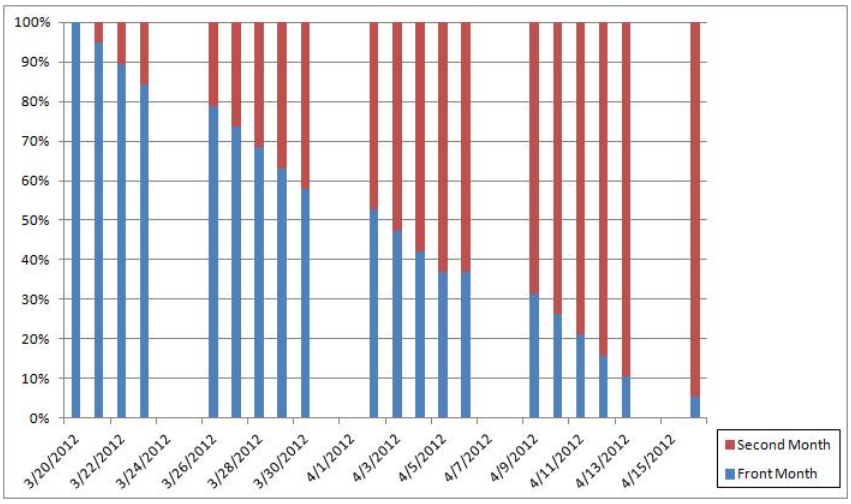

Figure 2: Front month vs. second month VIX futures contract weights within a roll period.

Based on (6), (7) in the preceding section, it can be seen that the weights calculations are such that the front month VIX futures will have its most influence on SPVIXSTR Index at beginning of the roll period and have its least influence on the last day of the roll period. What this means, practically speaking, is that because the correlation of the VIX index versus the individual VIX futures contracts decays with contract expiration, VXX will always attain its best[6] tracking potential versus the VIX on the first day of the roll period, begin to more significantly underperform from that point, and attain its worst tracking potential on the day before the next roll period. This "vicious cycle" repeats itself across roll periods and over the life of the SPVIXSTR Index and hence VXX. The positive yet imperfect correlation between the VIX Index and the first eight VIX futures contracts are presented in Figures 3 and 4. Correlation of the VIX futures curve vs. the VIX index is clearly non-perfectly positive and generally decays with contract maturity.

Security	VIX	UX1	UX2	UX3	UX4	UX5	UX6	UX7	UX8
VIX	1.000	0.838	0.814	0.841	0.847	0.834	0.836	0.843	0.813
UX1	0.838	1.000	0.938	0.919	0.898	0.857	0.847	0.834	0.829
UX2	0.814	0.938	1.000	0.962	0.940	0.911	0.903	0.874	0.870
UX3	0.841	0.919	0.962	1.000	0.966	0.949	0.938	0.926	0.906
UX4	0.847	0.898	0.940	0.966	1.000	0.964	0.956	0.950	0.928
UX5	0.834	0.857	0.911	0.949	0.964	1.000	0.968	0.968	0.951
UX6	0.836	0.847	0.903	0.938	0.956	0.968	1.000	0.972	0.961
UX7	0.843	0.834	0.874	0.926	0.950	0.968	0.972	1.000	0.976
UX8	0.813	0.829	0.870	0.906	0.928	0.951	0.961	0.976	1.000

Figure 3: Matrix based on daily returns from May 3, 2010 to May 3, 2012.

Security	VIX	UX1	UX2	UX3	UX4	UX5	UX6	UX7	UX8
VIX	1.000	0.852	0.800	0.785	0.771	0.735	0.742	0.729	0.508
UX1	0.852	1.000	0.916	0.876	0.857	0.794	0.800	0.783	0.616
UX2	0.800	0.916	1.000	0.968	0.946	0.913	0.912	0.888	0.670
UX3	0.785	0.876	0.968	1.000	0.961	0.946	0.940	0.929	0.697
UX4	0.771	0.857	0.946	0.961	1.000	0.961	0.960	0.951	0.739
UX6	0.742	0.800	0.912	0.940	0.960	0.973	1.000	0.978	0.780
UX5	0.735	0.794	0.913	0.946	0.961	1.000	0.973	0.973	0.731
UX7	0.729	0.783	0.888	0.929	0.951	0.973	0.978	1.000	0.797
UX8	0.508	0.616	0.670	0.697	0.739	0.731	0.780	0.797	1.000

Figure 4: Matrix based on weekly returns from May 3, 2010 to May 3, 2012.

Thus, due to the negative effects of contango and correlation relative to the construction of the SPVIXSTR Index, VXX essentially underperforms as both a viable long term investment holding and as an accurate proxy for trading and hedging volatility as defined by the VIX Index. Investor experience seems to conclude similarly[7]. The next section covers how one might attempt to capture the risk of holding a position in VXX.

[6] I'm using the word "best" loosely here.

[7] According to Richard Lehman's and Lawrence G. McMillan's book Options for Volatile Markets (2nd Ed.), an outright position in the VIX futures appears to be superior to VXX.

4 Risking VXX

In order to calculate the risk of loss in a VXX position it's important to understand what one actually has exposure to. A long position in VXX is comprised of the return arising from a long position in the SPVIXSTR Index plus the return from a long position in the NAV basis, or NAV Prem/Disc, defined as VXX price minus NAV price, divided by NAV price.

Starting with the first component, and as detailed previously, the SPVIXSTR Index return can further be decomposed into returns arising from 1) long positions in first month and second month VIX futures contracts, 2) a return on the cash position created from the daily futures roll including interest on interest. The following describes the situation:

$$\begin{aligned} VXXReturn(t) &= SPVXSTRIndexReturn(t) + NAVPrem/DiscReturn(t) \\ &= NAVReturn(t) + NAVPrem/DiscReturn(t) \\ &= CDR(t) + TBR(t) + NAVPrem/DiscReturn(t) \\ &\approx CDR(t) + NAVPrem/DiscReturn(t) \\ &\approx UX1Return(t) * UX1weight(t-1) + UX2Return(t) * UX2weight(t-1) \\ &\quad + NAVPrem/DiscReturn(t) \end{aligned} \tag{8}$$

where:
NAV Prem/Disc Return(t) = NAV Prem/Disc(i) - NAV Prem/Disc(i-1).
CDR(t)and TBR(t) are as per (2) and (5), respectively.
UX1 and UX2 are the continuously contracted front month and second month VIX futures contracts.
UX1 weight(t-1) and UX2 weight(t-1) are the corresponding lagged weights.

Data quality issues aside, equation (8) is visualized in Figure 5.

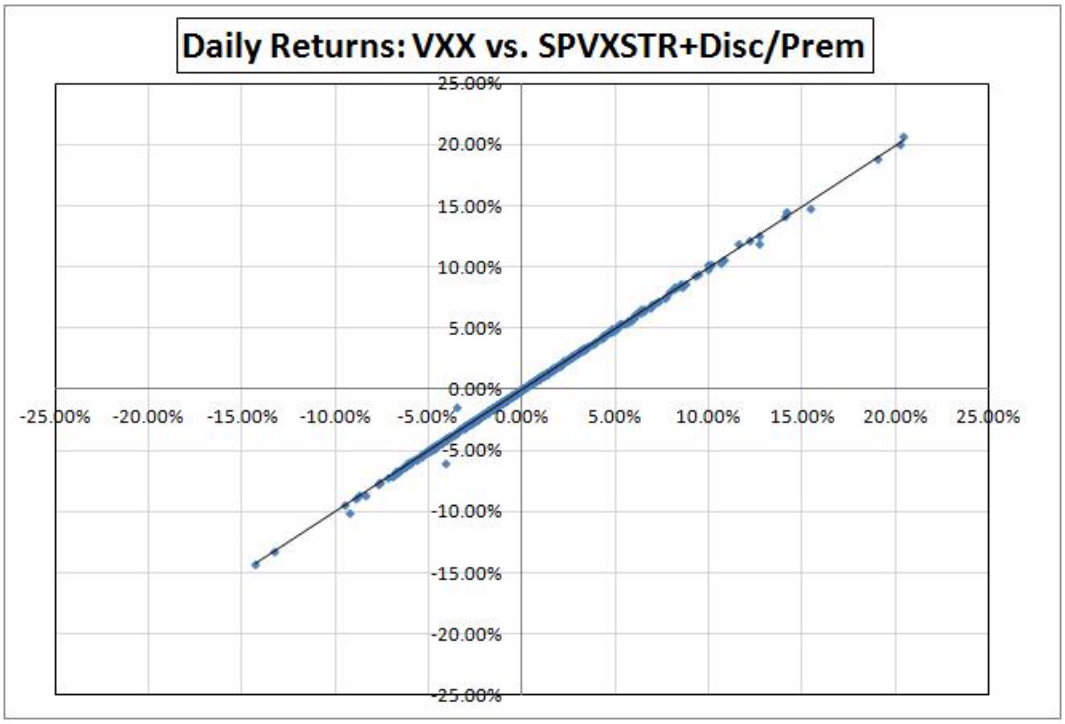

Figure 5:

As detailed in (5), the contribution to total return attributed to the daily futures roll can safely be ignored from a risk perspective, given its small contribution. In fact, the daily futures roll has historically only contributed around 74 basis points

per day to total return[8]. This small attribution can mainly be ascribed to the generally declining value of the SPVIXSTR index (driven by contango), and the presence of historically low interest rates since its introduction. Otherwise, the drivers of VXX return defined in (8) can be translated into corresponding risk factors and used in some type of Value-at-Risk (VaR) or stress model.

5 Conclusion

As a vehicle for trading volatility, the challenges with VXX are that: 1) it's futures-based and thus exposed to persistent contango, 2) while highly correlated with the VIX index, it does not exactly track exactly this volatility index which is of most interest to traders and hedgers. To see how imprecisely the VXX has tracked the VIX Index can be observed in Figure 6.

Figure 6: VXX vs. VIX Index over the longer term.

It can be seen that during the period closer to its debut on January 29, 2009, VXX tracked the VIX Index relatively well. However, the VXX did not closely trace the volatility spikes experienced in 2010 and in 2011. Over time, the ravages of contango and imperfect correlation catch up and can severely impact long term performance. As would be expected, the same situation affects the SPVIXSTR Index, as per Figure 7.

Figure 7: SPVIXSTR Index vs. VIX Index over the longer term.

As mentioned previously, VXX was designed for use as a short to medium-term trading vehicle. Therefore, underperform as both a vehicle for expressing views on volatility, and as a hedge for a long equity portfolio, is more likely if held over

[8]As one would expect, the return arising from the VIX futures contracts dominates. Only on days when VIX futures are relatively unchanged does the futures roll return dominate total return

a long time horizon.[9]. In other words, short to medium-term holding period horizons are expected to reduce the negative impacts of contango and imperfect correlation and allow VXX to perform better relative to the VIX index. However, these negative impacts can only be reduced, not eliminated. Buyer beware!

[9]Deng, Geng, McCann, Craig J. and Wang, Olivia, Are VIX Futures ETPs Effective Hedges? (June 27, 2012). Available at SSRN: `http://ssrn.com/abstract=2094624`