# Buy-Write or Put-Write,
# An Active Index Writing Portfolio to Strike it Right

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## Abstract

Can simple technical analysis add value to index option selling investment strategies? To test this, I propose a dynamic allocation approach to construct option writing portfolios. Unlike the standard passive Buy-Write (covered call) and Collateralized Put-Write strategies, an *active leveraged option overlay portfolio* (*ALOOP*) involve switching between shorting call and writing put index options – a market timing scheme based on a technical rule.

In particular, I use S&P 500 Total Return Index as the underlying equity index and choose a simple but well known double moving average cross rule – Golden Cross/Black Cross as the trading signal. The portfolio is expressed analytically with two parameters controlling the level of portfolio leverage for an expected bullish or bearish market regime, respectively. In a back-test with 22.6 years (from 06/01/1988 to 12/31/2010) of daily close data, I consider different levels of estimated transaction costs for monthly portfolio rebalance, index option contracts roll-over or settlement, and active trading. An example case of the active portfolio achieves better returns and risk-adjusted returns than the CBOE S&P 500 Monthly Buy-Write (*BXM*) or Put-Write (*PUT)* Index, and out-performs the underlying equity index by over 5% in annualized return with similar levels of risk.

I further introduce a model based estimation of the active portfolio's Greek Letter *Delta.* It is shown as the basis of an effective trading rule to improve portfolio performance by managing market risk weekly. Other special cases demonstrate that the dynamic overlay of written index options can be used as an alpha generating tool for cash management and passive equity index investments. A return attribution of the active investment portfolio identifies an active alpha from the Golden Cross/Black Cross market timing, a volatility skew risk premium, and their positive interactions. Potential market impacts of the active index writing strategies are also discussed.

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## 1. Introduction: Buy-Write, Put-Write and Beyond

Stock market predictability, portfolio allocation and derivative pricing, are three prominent topics in modern finance. Besides ample academic and empirical research, there are great practical interests to develop and implement investment strategies reflecting these theoretical underpinnings. With the complex and constantly changing financial markets, is it possible to design a time tested investment scheme that addresses all three areas in formulation, takes advantage of their interactions, and yet appeals to investors with feasibility and simplicity?

As simple passive portfolio strategies, “Buy-Write” (covered call option writing) and “Put-Write” (cash collateralized shorting of put option) had been applied to broad stock market indexes, notably, *S&P 500 Index* (*SPX*) as a first. In 2002, *Chicago Board of Option Exchange* (*CBOE*) introduced *S&P 500 Monthly Buy-Write Index* (*BXM*) which can be used as a performance benchmark for related mutual funds, exchange-traded-funds (ETF) and other investment products. *BXM* was examined by Whaley (2002) and Feldman and Roy (2005) regarding its performance characteristics, invest-ability and the sources driving the performance. The *CBOE S&P 500 Monthly Put-Write Index* (*PUT*) was launched in 2007, followed by Ungar and Moran (2009)’s detailed analysis. By capitalizing on a negative volatility risk premium (Bakshi and Kapadia, 2003) through mechanical index option writing, standard Buy-Write and Put-Write strategies achieved on average better returns and reduced risks compared to the underlying S&P 500 Index[1].

However, it should also be emphasized that Buy-Write and Put-Write strategies under-performed the underlying equity index in strong rising markets such as the 1990’s, and that they could not avoid heavy losses during precipitous market declines, most recently in the 2008-2009 market crisis. The Buy-Write and Put-Write strategies use 100% fixed allocations in their simple portfolio to fully cover or collateralize written index option positions. In other words, regardless any fundamental or technical market predictions, they always target at a constant portfolio beta (around one half for *BXM* and *PUT* in nominal term) to cause under-exposure in a bull market, and over-exposure (compared to that of risk free assets) in a bear market. To alleviate the underperformance of Buy-Write strategy in a rising market, written out-of-money (OTM) call options (over-writing) can be used to reduce the chances of exercise in the money, but usually with the trade-off of collecting less call premium. The *CBOE S&P 500 2% OTM Buy-Write Index* (*BXY*)[2] is an index of passive over-writing that exhibited better risk-adjusted return than *BXM.*

Hill et al (2006) considered a dynamic strike modification to the over-writing Buy-Write strategy based on on-going volatility environment. A jump in the implied volatilities of index options generally accompanies a market drop, and vice versa. Targeting at a fixed probability of index options expiring in-the-money, larger implied volatility raises the out-of-money level of strike prices in the written calls and reduces the call premium collected. Thus compared to the fixed call over-writing strategies, the market timing of dynamic strike generally can improve an over-writing strategy's performance only when the market's rising or falling short term trend reverses by the option expiration day of the coming month.

Recent practice as variations of Buy-Write strategies also introduced market timing, for example, to adjust the strike price or out-of-money-ness of the written call options using technical analysis rules of simple moving average (*SMA*) cross. By predicting a bullish or bearish market trend from 200-day *SMA* cross of S&P 500 index, Baehr (2010) used at-the-money call writing in bearish market regime and call over-writing in a bullish regime to demonstrate significant performance improvement over standard Buy-Write strategies[3].

Nevertheless, from a portfolio perspective, the modified buy-write formulations with a technical analysis based market timing scheme have not yet addressed two important aspects:

First, using index put option writing rather than index call writing when expecting an upward market trend[4]. When predicting market to rise in short term by next monthly index option expiration, the current at-the-money (ATM) index put option is expected to be less probable to expire in the money than the current at-the-money call option. ATM index put writing out-performs the ATM index call writing should the bullish market prediction turn out to be true.

Second, varying the portfolio's overall equity market exposure according to a market timing scheme, i.e., not only changing the portfolio's written index option position (on put or call, strike price, etc) – but also modify portfolio weightings in underlying equity index and risk free bond accordingly. This essentially proposes dynamic asset allocation among portfolio components of underlying equity index, index options and risk free assets.

Taking advantage of stock market predictability and consequent time varying investment opportunities, dynamic asset allocation should be used for a simple portfolio of stock index and risk free asset (see Kim and Omberg 1996, Kandel & Stambaugh 1996, Xia 2001and Detemple, Garcia & Rindisbacher 2003) to out-perform a passive optimal portfolio from modern portfolio theory. In particular, Zhu and Zhou (2009) focused on technical analysis rules of moving average as the market timing system for dynamic asset allocation in stock (S&P 500 index) and risk-less bond. They found that, with a simple moving average cross signal, active two-state constant adjustment to the fixed optimal portfolio weights can improve portfolio performance substantially. This theoretical formulation of dynamic asset allocation has two important aspects suitable for practical implementation:

First, portfolio performance from dynamic allocations switching between two sets of fixed weights for stock market index and risk-less bond, is more robust than the continuous optimal dynamic allocations when there are significant uncertainties or errors in modeling of stock market. The two sets of fixed weights can be specified through historical learning and optimization.

Second, under reasonable long investment time horizon (e.g. ~ 20 years), the optimal lag for moving average turns out much greater than 200 days for S&P 500 Index (*SPX*). As a result, the moving average crosses that trigger market timing trades or portfolio rebalance, are not too frequent to impact performance negatively due to transaction costs and market friction.

To extend the technical analysis based dynamic portfolio allocation approach to include index option writing, I will introduce an empirical model using *S&P 500 Total Return Index* (*SPTR*) as the underlying stock index investment. Rather than using data of specific *SPX* put or call options, *BXM* and *PUT* Indexes are used as building blocks in the portfolio construction to reflect invest-ability[5].

The paper is structured as the follows: Section 2 introduces a market-timing scheme to test simple tactical allocation portfolios that use *BXM*, *BXY* or *PUT* index to replace *SPTR*. Section 3 sets up an *SPTR* / *SPX* based dynamic index option writing portfolio model that includes two parameters for portfolio leverage, and uses *BXM* and *PUT* indexes as building blocks for option components. Several special cases of the proposed active portfolio are examined in Section 4 to demonstrate dynamic index writing as an alpha generating tool for cash management and passive index investments. Based on an analytical estimation of portfolio *Delta*, Section 5 looks at the risk management aspects of the active portfolio and tests a specific weekly trading scheme to improve performance. Portfolio *Delta* estimation also helps to define an *active benchmark portfolio* and attributing the excess returns into an active benchmark alpha, volatility skew risk premium and their interactions. Section 6 discusses the potential market impact of the active index writing scheme, the long term profit sustain-abilities of proposed portfolio, and concludes.

## 2. Golden Cross Market Timing with Buy-Write or Put-Write

Moving Average is one of the most versatile and widely used technical analysis tools for active investment management (Murphy, 1997). First applied in commodities futures trading then for equity index (Brock, Lakonishok & Lebaron, 1992), price moving-average cross signal is the basis of most trending-following system today. Smoothing prices with a time lag, the premise of moving average trend following system is that price momentum exists at certain time scale. Zhu and Zhou (2009) showed that the optimal time lag for simple moving average stock market trading system could be related to the time scale of a fundamentals based stochastic market model.

Due to uncertainties associated with the theoretical market models, time lag in price moving average is usually determined in practice by empirical analysis or through historical back testing. Trend following using longer range moving average becomes advantageous as they can avoid minor corrections or consolidations and ride with the major trend longer. However, it can have more delay in responding to trend reversal.

Market timing with equity index option writing can also be put in the context of a trend following system based on underlying index price moving average. Call writing has a short market exposure which should be used when a down trend is identified from a moving average system. On the other hand, index put writing has a long exposure to market and is thus suitable when index price moving average indicates an up trend.

Besides option price premium related to market momentum (Amin, Coval & Seyhun (2004)), there are directional synergies between moving average trading signal and the index option writing strategy. In trending periods correctly signaled by the moving average, 100% of index option premium become additional income as option expiring in-the-money is avoided. Even when the moving average signal is less successful due to delayed response to trend change, option premium collected can offset a portion of the losses in the underlying equity position. Using moving average with longer time lag can also avoid excessive trading. In a trend-less market, market timing trading intervals that are much longer than a month can take full advantage of the negative *Theta* of monthly expiring options, such as those in *BXM* and *PUT* indexes.

As a popular "double cross-over method", Golden Cross / Black Cross turned out satisfying the requirement of an active index writing scheme on the broad market index such as S&P 500 Total Return Index (*SPTR*). Golden Cross refers to 50 Day Simple (arithmetic) Moving Average (50 DMA) crossing above 200 Day Simple Moving Average (200 DMA), while Black Cross refers to 50 DMA crossing below 200 DMA.

Table 1 shows that during the period of 22.6 years (6/1/1988 to 12/31/2010), Golden Cross and Black Cross on *SPTR* happened alternately for a total of 21 times. The longest bullish period (from Golden Cross to Black Cross triggers) lasted 1032 trading days (8/31/1994 to 10/1/1998), while the shortest bullish period only lasted a month (21 trading days between 4/19/2002 and 5/20/2002). The *SPTR* period returns between Golden Cross and Black Cross trigger days are also listed in Table 1. For bullish period, the Golden Cross signal is indicated as "right" if the period return following the signal (until the next Black Gross triggers) is positive, and "wrong" if otherwise; for bearish period, the Black Cross signal is indicated as "right" if the period return following the signal (until the next Golden Cross triggers) is negative, and "wrong" if otherwise.

## Table 1: S&P 500 Total Return Index (*SPTR*)'s Golden Crosses and Black Crosses* (6/1/1988-12/31/2010)

| Signal Trade Date | Type of Signal | Duration Till Next Signal (Trading Days) | SPTR P/L Till Next Signal | Bullish/Bearish Signal Right or Wrong |
|---|---|---|---|---|
| 6/16/1988 | Golden Cross (Bullish) | 439 | 32.42% | Right |
| 3/13/1990 | Black Cross (Bearish) | 42 | 5.32% | Wrong |
| 5/11/1990 | Golden Cross (Bullish) | 88 | -8.61% | Wrong |
| 9/17/1990 | Black Cross (Bearish) | 101 | 14.81% | Wrong |
| 2/8/1991 | Golden Cross (Bullish) | 815 | 38.57% | Right |
| 5/2/1994 | Black Cross (Bearish) | 85 | 6.07% | Wrong |
| 8/31/1994 | Golden Cross (Bullish) | 1032 | 126.25% | Right |
| 10/1/1998 | Black Cross (Bearish) | 45 | 19.61% | Wrong |
| 12/4/1998 | Golden Cross (Bullish) | 484 | 24.06% | Right |
| 11/3/2000 | Black Cross (Bearish) | 361 | -19.60% | Right |
| 4/19/2002 | Golden Cross (Bullish) | 21 | -2.81% | Wrong |
| 5/20/2002 | Black Cross (Bearish) | 246 | -11.93% | Right |
| 5/12/2003 | Golden Cross (Bullish) | 326 | 19.59% | Right |
| 8/26/2004 | Black Cross (Bearish) | 43 | 2.11% | Wrong |
| 10/27/2004 | Golden Cross (Bullish) | 439 | 16.39% | Right |
| 7/26/2006 | Black Cross (Bearish) | 24 | 3.06% | Wrong |
| 8/29/2006 | Golden Cross (Bullish) | 335 | 16.27% | Right |
| 12/28/2007 | Black Cross (Bearish) | 370 | -35.55% | Right |
| 6/18/2009 | Golden Cross (Bullish) | 263 | 14.32% | Right |
| 7/6/2010 | Black Cross (Bearish) | 70 | 15.26% | Wrong |
| 10/13/2010 | Golden Cross (Bullish) | 55 | 7.21% | Right |

*The duration and profit/loss of the last Golden Cross (10/13/2010) is only counted up to 12/31/2010. Daily close values of *SPTR* are used.

## Table 2: Effectiveness of *SPTR* Golden Crosses and Black Crosses* (6/1/1988-12/31/2010)

| | SPTR Golden Crosses (Times) | SPTR Golden Crosses (Sum of SPTR P/L) | SPTR Golden Cross Average P/L | SPTR Black Crosses (Times) | SPTR Black Crosses (Sum of SPTR P/L) | SPTR Black Cross Average P/L |
|---|---|---|---|---|---|---|
| **Right** | 9 | 295.07% | 32.79% | 3 | -67.08% | -22.36% |
| **Wrong** | 2 | -11.42% | -5.71% | 7 | 66.25% | 9.46% |

*Partial duration and profit/loss of the last Golden Cross (10/13/2010) from Table 1 is used in Table 2's total and average *SPTR* Profit/Loss calculations. Sum of each rising period returns following a Golden Cross is divided by the total number of such periods to get the average Profit/Loss of a "right" *SPTR* Golden Cross. Sum of each falling period returns following a Golden Cross is divided by the total number of such periods to get the average P/L of a "wrong" *SPTR* Golden Cross. Average *SPTR* P/L of "Right" or "Wrong" Black Crosses is similarly defined.

Table 2 indicates the effectiveness of the *SPTR* Golden Cross and Black Cross signals. Golden Cross signals were right nine out of eleven times and a right Golden Cross bullish signal were historically ensued by over-whelming upside (32.8% gain on average) compared to a wrong

Golden Cross signal's downside move (an average 5.7% loss). Black Cross signals, though right only three out of ten times, historically can avoid bear market (on average 22.4% loss) every time it is right. Seven wrong Black Cross signals miss on average a market gain each time of 9.5%. The delay in responding to a recovery from bear market seems more serious than delay in exiting a long position before a major market decline. The problematic Black Cross signals were on 9/17/1990, 10/1/1998 (the worst) and 7/6/2010, that during the subsequent 101, 45 and 70 trading sessions (till a next Golden Cross) *SPTR* actually gained 14.8%, 19.6% and 15.3%, respectively.

**Table 3: *SPTR* Monthly* Performance Statistics**
**Following Golden Crosses and Black Crosses (6/1/1988-12/31/2010)**

| | SPTR Golden Crosses (Number of Full Option Months) | SPTR Golden Crosses (SPTR Average Monthly P/L) | SPTR Golden Cross Best / Worst Monthly P/L and Time | | SPTR Black Crosses (Number of Full Option Months) | SPTR Black Crosses (SPTR Average Monthly P/L) | SPTR Black Cross Best/Worst Monthly P/L and Time | |
|---|---|---|---|---|---|---|---|---|
| **Right** | 125 | 3.09% | 14.14% | Oct.-Nov. 1999 | 25 | -6.07% | -24.95% | Sep.-Oct. 2008 |
| **Wrong** | 70 | -2.20% | -8.99% | July-Aug. 1990 | 29 | 4.45% | 13.33% | Mar.-Apr. 2009 |

*Monthly returns are calculated using daily close values of *SPTR* from the *SPX* option expiration Friday of the current month to the option expiration Friday the following month – a same approach taken in Hill, *et al* (2006). For example, the Sep.-Oct. 2008 month *SPTR* return is calculated with close prices on 9/19/2008 and 10/17/2008. Partial months due to a Golden Cross or Black Cross triggering, and also the partial month 12/17/2010 to 12/31/2010, are excluded from the statistics.

It has been noticed that all the two wrong Golden Cross and seven wrong Black Cross signals have *ex post* duration less than 101 days; and the right signals all lasted for at least about a year (246 trading days). Overall, the Golden Cross and Black Cross market timing system is effective in capturing most of the rising market and avoiding severe market declines. Relevant to index option writing strategies, it is also of interest to examine the monthly performance of *SPTR* (see Table 3) following the Golden Cross and Black Cross signals. The *SPTR* monthly returns after a Black Cross appeared to be more volatile than those after a Golden Cross signal. After a Black Cross signal, *SPTR* index declined in less than half of the months (25 out of 54 months). However, the average monthly decline following a Black Cross is more serious than the average monthly gain, and using Black Cross signal would have avoided a disastrous month such as Sep. – Oct. 2008. Periods following a Golden Cross signal had a historical rate at 64% (125 months out of 195) that *SPTR* index rose. On average, the monthly *SPTR* gain in a rising month following a Golden Cross is more than the average monthly loss of the decline months after a Golden Cross.

Golden Cross/Black Cross (*GCBC*) signal using daily close prices of S&P 500 Total Return Index (SPTR) can be implemented as a simplest tactical portfolio $\Pi_{SPTR}$ (called *SPTR GC-LEO*: S&P 500 Total Return Index Golden Cross Long Equity Only strategy) [6]:

$$\Pi_{SPTR} = \begin{cases} SPTR, & when \quad SPTR \quad 50DMA \geq 200DMA \\ B, & when \quad SPTR \quad 50DMA < 200DMA \end{cases} \tag{1}$$

where *B* represents the position invested in a risk free asset, e.g. the 3-month US Treasury Bills.

To combine index option writing strategies with the *SPTR* Golden Cross /Black Cross signal, simply replace the *SPTR* position in Equation (1) of *SPTR-GC-LEO* strategy with an option writing index such as *BXM*, *PUT* or *BXY*, and call the alternative strategies as *SPTR-GC-BXM*, *SPTR-GC-PUT* and *SPTR-GC-BXY*, respectively. In periods following a Golden Cross signal when *SPTR-GC-LEO* has long *SPTR* position, the alternative strategies settle expiring index options, write new monthly index options and rebalance portfolio into *SPTR* or Treasuries Bills, all on option expiring Fridays – just as the respective option writing index does. On the other hand, in the expected bearish periods following a Black Cross, all three alternative strategies hold 3-month T-Bills positions – the same as the *SPTR-GC-LEO* strategy.

**Table 4: Indexes and Golden Cross Technical Strategies Performance Metrics Comparison (6/1/1988-12/31/2010)**

| | SPTR | BXM | PUT | BXY | SPTR GC-LEO | SPTR GC-BXM | SPTR GC-PUT | SPTR GC-BXY |
|---|---|---|---|---|---|---|---|---|
| **Annualized Return** | 9.52% | 9.70% | 11.02% | 10.69% | 11.01% | 10.28% | 11.24% | 11.23% |
| **Annualized Std Deviation** | 18.25% | 13.06% | 11.90% | 14.57% | 12.50% | 8.96% | 7.74% | 10.15% |
| **Sharpe Ratio** | 0.294 | 0.426 | 0.577 | 0.450 | 0.549 | 0.685 | 0.917 | 0.698 |
| **Sortino Ratio** | 0.421 | 0.591 | 0.799 | 0.628 | 0.790 | 0.964 | 1.283 | 0.982 |
| **Skew** | -0.039 | -0.271 | -0.326 | -0.314 | -0.327 | -0.389 | -0.888 | -0.636 |
| **Kurtosis** | 9.189 | 26.460 | 29.338 | 14.467 | 6.752 | 40.882 | 24.057 | 10.530 |
| **Alpha** | 0.00% | 2.15% | 3.80% | 2.48% | 4.34% | 4.61% | 5.76% | 5.15% |
| **Beta** | 1.000 | 0.635 | 0.571 | 0.757 | 0.469 | 0.284 | 0.250 | 0.360 |
| **Stutzer Index (Daily)** | 2.670E-04 | 4.249E-04 | 7.018E-04 | 4.779E-04 | 6.490E-04 | 9.193E-04 | 1.542E-03 | 9.605E-04 |
| **Stutzer Equivalent Sharpe Ratio** | 0.367 | 0.463 | 0.595 | 0.491 | 0.572 | 0.681 | 0.882 | 0.696 |
| **Maximum Drawdown** | 55.25% | 40.14% | 37.09% | 44.83% | 19.19% | 15.36% | 14.36% | 16.90% |
| **Ending Value Multiple** | 7.803 | 8.111 | 10.610 | 9.937 | 10.589 | 9.137 | 11.113 | 11.083 |

*All performance metrics are calculated from daily return data due to the daily nature of the Golden Cross signal. The risk free rate used is the average annualized yield of 3-month T-bills from 06/01/1988 to 12/31/2010 at 4.15%. The "Stutzer Equivalent Sharpe Ratio" is calculated by $\sqrt{252 \times 2 \times SI}$ where *SI* is the Stutzer Index calculated from daily return stream. "Ending Value Multiple" in the last row of the table is the equity curve value of the portfolio on 12/31/2010 assuming starting portfolio value at 1 on 6/1/1988 without any tax, payout, withdrawal or additions.

For period of 22.6 years (6/1/1988-12/31/2010), Table 4 shows the performance metrics of the *SPTR*, three passive option writing indexes and four tactical option overlay strategies. Table 5 shows returns of these strategies year by year. The performance metrics of *SPTR-GC-LEO* indicated effectiveness of the Golden Cross signal. *SPTR-GC-LEO* had better average annual return than the *SPTR* with less than 70% of the standard deviation of *SPTR* and a maximum drawdown within 20%. Despite the largest negative skew and high kurtosis in daily return distribution, *SPTR GC-PUT* stood out as the best tactical allocation strategy, outperforming *SPTR* index by 1.7% in annualized return with the lowest risk level at only 42% of *SPTR*'s annualized standard deviation, a beta of 0.25 and a maximum drawdown within 15%. *SPTR GC-PUT* also achieved the best risk adjusted returns, as measured in Sharpe ratio, Sortino ratio, and the Stutzer Index (Stutzer 2000) which penalizes the negative skew common in option based strategies. *SPTR GC-BXY* did not out-perform *SPTR GC-PUT* in any performance metrics over the back-tested period. Thus an at-the-money (ATM) put-write is chosen, rather than the out-of-money (OTM) call-write, as a building block for the portfolio model in the next section.

**Table 5: Indexes and SPTR Golden Cross Tactical Strategies**
**Year by Year Returns (6/1/1988-12/31/2010)**

| | SPTR | BXM | PUT | BXY | SPTR GC-LEO | SPTR GC-BXM | SPTR GC-PUT | SPTR GC-BXY |
|---|---|---|---|---|---|---|---|---|
| **1988*** | 6.33% | 8.13% | 6.90% | 9.76% | 5.28% | 6.36% | 6.90% | 8.46% |
| **1989** | 31.69% | 25.01% | 24.58% | 32.58% | 31.69% | 25.01% | 24.58% | 32.58% |
| **1990** | -3.10% | 3.99% | 8.88% | 1.93% | -9.41% | -2.28% | 1.86% | -5.08% |
| **1991** | 30.47% | 24.39% | 21.32% | 22.93% | 20.27% | 20.16% | 16.75% | 18.38% |
| **1992** | 7.62% | 11.52% | 13.80% | 11.04% | 7.62% | 11.52% | 13.80% | 11.04% |
| **1993** | 10.08% | 14.10% | 14.14% | 11.02% | 10.08% | 14.10% | 14.14% | 11.02% |
| **1994** | 1.32% | 4.50% | 7.10% | 4.60% | -3.05% | 2.43% | 5.50% | -0.05% |
| **1995** | 37.58% | 20.97% | 16.88% | 33.20% | 37.58% | 20.97% | 16.88% | 33.20% |
| **1996** | 22.96% | 15.50% | 16.40% | 19.83% | 22.96% | 15.50% | 16.40% | 19.83% |
| **1997** | 33.36% | 26.64% | 27.68% | 29.75% | 33.36% | 26.64% | 27.68% | 29.75% |
| **1998** | 28.58% | 18.95% | 18.54% | 21.24% | 8.32% | 7.59% | 8.76% | 6.52% |
| **1999** | 20.78% | 21.40% | 20.98% | 19.75% | 20.78% | 21.40% | 20.98% | 19.75% |
| **2000** | -9.10% | 7.40% | 13.06% | 1.96% | -1.03% | 9.97% | 13.26% | 6.85% |
| **2001** | -11.89% | -10.92% | -10.63% | -11.41% | 3.61% | 3.61% | 3.61% | 3.61% |
| **2002** | -22.10% | -7.64% | -8.58% | -12.25% | -1.34% | 0.72% | 1.25% | -0.35% |
| **2003** | 28.68% | 19.37% | 21.77% | 24.91% | 19.53% | 12.67% | 14.73% | 15.36% |
| **2004** | 10.88% | 8.30% | 9.48% | 9.74% | 8.92% | 6.92% | 8.36% | 7.78% |
| **2005** | 4.91% | 4.25% | 6.71% | 4.41% | 4.91% | 4.25% | 6.71% | 4.41% |
| **2006** | 15.79% | 13.33% | 15.16% | 17.14% | 12.90% | 12.27% | 13.88% | 15.13% |
| **2007** | 5.49% | 6.59% | 9.51% | 6.11% | 6.25% | 6.95% | 9.90% | 6.66% |
| **2008** | -37.00% | -28.65% | -26.77% | -31.23% | 1.43% | 1.43% | 1.43% | 1.43% |
| **2009** | 26.46% | 25.91% | 31.51% | 32.07% | 22.89% | 17.20% | 17.95% | 21.25% |
| **2010** | 15.06% | 5.86% | 9.02% | 9.82% | -0.13% | -5.37% | -5.10% | -2.53% |

* Partial year return is used for 1988 (06/01/1988 to 12/31/1988).

## 3. Construction of *Active Leveraged Option Overlay Portfolio* (*ALOOP*)

Based on Golden Cross / Black Cross signals over an underlying equity index $S$, a two-state portfolio is proposed that writes ATM put options ($P$) in an expected bullish period or ATM call options ($P$) in an expected bearish period. The number of option contracts makes the ATM options' initial face value equal the portfolio's total value[7]. Introducing two parameters of leverage, $F_L$ and $F_S$, for the periods following a Golden Cross and a Black Cross respectively, an Active Leveraged Option Overlay Portfolio (*ALOOP*), $\Pi$, comprised of equity index $S$, written European index call or put options, $C$ or $P$, and a risk free asset (e.g. 3-month T-Bill) $B$, can be expressed as[8]:

$$\Pi = \begin{cases} F_L S + (1 - F_L)B - P, & When \quad 50DMA \geq 200DMA \quad on \quad S \\ (1 - F_S)S + F_S B - C, & When \quad 50DMA < 200DMA \quad on \quad S \end{cases} \tag{2}$$

where the notation of the $F_L\ S$ and $F_S\ B$ terms means $F_L$ and $F_S$ portion of the portfolio total value including option premium collected, are invested in stock index and T-bills respectively.

When Golden Cross signal triggers, the trading transactions are: swapping written call options with writing put options, and exchanging $(1 - F_L - F_S)$ portion of the portfolio value out of stock index $S$ holdings, which totaled $(1 - F_S)$ portion of portfolio value, to T-bills $B$. When Black Cross signal triggers, the portfolio transactions are: buy to cover all written put options; sell call option contracts; and liquidate stock index positions, which totaled $F_L$ portion of portfolio value, by amount of $(F_L + F_S - 1)$ portion of the portfolio value to invest in T-bills.

When $F_L + F_S = 1$, there is no exchange trades between stock index and T-bills on signal triggering day. Rebalance due to market movement since last option expiration is possible.

The meaning of *Factor of Leverage* $F_L$ and $F_S$ are clearer by introducing the standard Collateralized Put-Write strategy ($CPW = B - P$) and Covered Buy-Write ($CBW = S - C$) strategy as building blocks, then rewrite equation (2) as:

$$\Pi = \begin{cases} CPW + F_L(S - B), & When \quad 50DMA \geq 200DMA \quad on \quad S \\ CBW - F_S(S - B), & When \quad 50DMA < 200DMA \quad on \quad S \end{cases} \tag{3}$$

In an expected bullish period following a Golden Cross, $F_L$ is thus the level of long leverage of a self-financed pair of long stock index and short T-bills, in addition to a fully collateralized put-write strategy. Similarly, $F_S$ is the level of short leverage of the self-financed pair of long stock index and short T-bills, in addition to a covered buy-write strategy in an expected bearish period following a Black Cross.

In order to evaluate the portfolio's value and underlying equity exposure (*Delta*) using a continuous option pricing theory, the portfolio model of equation (2) can assume continuous rebalance among three components: $S$, $B$, and $P$ or $C$,. However, to use option prices implied in monthly option writing indexes like *BXM* and *PUT* when the underlying equity index is *SPTR*, it is more practical to rebalance only on monthly option expiring Fridays and on Golden Cross/Black Cross triggering days. For the Golden Cross/Black Cross signal trigger day rebalance, the shares in stock index, T-bills and number of option contracts are allocated as if they evolved from the rebalanced levels of last monthly option expiration Friday to the signal trigger day. Set the stock index to *SPTR*, the Golden Cross/Black Cross Active Leverage Option Overlay Portfolio (*GCBC–ALOOP*) of equation (3), $\Pi_{SPTR}$, can be written as:

$$\Pi_{SPTR} = \begin{cases} PUT + F_L(SPTR - B), & When \quad 50DMA \geq 200DMA \quad on \quad SPTR \\ BXM - F_S(SPTR - B), & When \quad 50DMA < 200DMA \quad on \quad SPTR \end{cases} \tag{4}$$

For performance tracking purpose, this indicates that *SPTR GCBC-ALOOP* can be constructed from four component indexes: *SPTR, BXM, PUT* and three-month T-bills *B*. At time $t$ day close, the daily return of the *SPTR GCBC-ALOOP* since the previous day $t-1$ close is:

$$R_t = \begin{cases} \dfrac{(PUT_t - PUT_{t-1}) + F_L(SPTR_t - SPTR_{t-1}) - F_L(B_t - B_{t-1})}{\Pi_{SPTR}}, & when \quad SPTR \quad 50DMA \geq 200DMA \\ \dfrac{(BXM_t - BXM_{t-1}) - F_S(SPTR_t - SPTR_{t-1}) + F_S(B_t - B_{t-1})}{\Pi_{SPTR}}, & when \quad SPTR \quad 50DMA < 200DMA \end{cases} \tag{5}$$

where $\Pi_{SPTR}$ is portfolio value at time $t-1$ day close from equation (4).

## 4. Performance of *SPTR GCBC-ALOOP* and Special Cases

The back-test of *SPTR GCBC-ALOOP* starts from 6/1/1988, when *PUT* and *BXM* started both with nominal value of 100, and ends on 12/31/2010. On Golden Cross or Black Cross days, *SPTR GCBC-ALOOP* switches between one share of *PUT* and *BXM*, and between long $F_L$ share and short $F_S$ share of self-financed pair position (*SPTR – B*). There are 21 Golden Cross / Black cross signal days during the back-tested 22.6 year period. Besides the same monthly option contracts roll-over and portfolio rebalance scheme used in *BXM* or *PUT*, it leads to 20 additional trades as one Golden Cross day (4/19/2002) happened to be an option expiration Friday.

Due to about 20% initial margin requirement[9] on writing *SPX* options, the *Factor of Leverage* $F_L$ is capped at 0.8. Imposing a constraint of no net short position in the stock index, both $F_L$ and $F_S$ have a lower limit of zero and $F_S$ has an upper limit of one. The whole range of two-parameter *Factor of Leverage* space ($F_L$, $F_S$) are tested with an increment of 0.1 each, to compare the *ex post* performance for the entire 22.6 year time span. This is different from the approach of Zhu and Zhou (2009) which derived the optimal fixed allocations for the portfolio given a utility function and investor time horizon. Table 6 listed annualized returns for all $F_L$ and $F_S$ combination considered. Table 7 – 9 compares the risk adjusted returns of Sharpe Ratio, Sortino Ratio and Stutzer Index (converted to equivalent annualized Sharpe Ratio) respectively.

**Table 6: Annualized Return of *SPTR GCBC-ALOOP* (6/1/1988-12/31/2010)**

| $F_L \setminus F_S$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 10.66% | 10.83% | 10.98% | 11.11% | 11.23% | 11.32% | 11.39% | 11.45% | 11.48% | 11.50% | 11.49% |
| 0.1 | 11.37% | 11.55% | 11.69% | 11.83% | 11.94% | 12.04% | 12.11% | 12.16% | 12.20% | 12.21% | 12.21% |
| 0.2 | 12.07% | 12.25% | 12.42% | 12.56% | 12.65% | 12.74% | 12.82% | 12.87% | 12.91% | 12.92% | 12.92% |
| 0.3 | 12.77% | 12.95% | 13.12% | 13.26% | 13.39% | 13.50% | 13.55% | 13.57% | 13.61% | 13.62% | 13.62% |
| 0.4 | 13.46% | 13.64% | 13.81% | 13.95% | 14.08% | 14.19% | 14.28% | 14.36% | 14.45% | 14.32% | 14.31% |
| 0.5 | 14.13% | 14.32% | 14.49% | 14.63% | 14.76% | 14.87% | 14.97% | 15.04% | 15.10% | 15.14% | 15.17% |
| 0.6 | 14.81% | 14.99% | 15.16% | 15.31% | 15.44% | 15.55% | 15.64% | 15.72% | 15.78% | 15.82% | 15.84% |
| 0.7 | 15.47% | 15.66% | 15.82% | 15.97% | 16.10% | 16.22% | 16.31% | 16.39% | 16.44% | 16.49% | 16.51% |
| 0.8 | 16.12% | 16.31% | 16.48% | 16.63% | 16.76% | 16.87% | 16.97% | 17.04% | 17.10% | 17.14% | 17.17% |

**Table 7: Sharpe Ratio of *SPTR GCBC-ALOOP* (6/1/1988-12/31/2010)**

| $F_L \setminus F_S$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5314 | 0.5893 | 0.6500 | 0.7108 | 0.7674 | 0.8138 | 0.8435 | 0.8519 | 0.8379 | 0.8051 | 0.7592 |
| 0.1 | 0.5563 | 0.6122 | 0.6634 | 0.7178 | 0.7660 | 0.8046 | 0.8291 | 0.8364 | 0.8258 | 0.7996 | 0.7616 |
| 0.2 | 0.5746 | 0.6256 | 0.6762 | 0.7238 | 0.7586 | 0.7919 | 0.8123 | 0.8187 | 0.8107 | 0.7897 | 0.7584 |
| 0.3 | 0.5876 | 0.6339 | 0.6787 | 0.7202 | 0.7557 | 0.7830 | 0.7972 | 0.8008 | 0.7947 | 0.7777 | 0.7518 |
| 0.4 | 0.5965 | 0.6382 | 0.6780 | 0.7142 | 0.7448 | 0.7682 | 0.7828 | 0.7879 | 0.7913 | 0.7649 | 0.7432 |
| 0.5 | 0.6021 | 0.6398 | 0.6752 | 0.7069 | 0.7335 | 0.7537 | 0.7665 | 0.7712 | 0.7680 | 0.7574 | 0.7501 |
| 0.6 | 0.6052 | 0.6392 | 0.6708 | 0.6988 | 0.7221 | 0.7398 | 0.7510 | 0.7555 | 0.7532 | 0.7445 | 0.7303 |
| 0.7 | 0.6063 | 0.6371 | 0.6654 | 0.6903 | 0.7109 | 0.7265 | 0.7366 | 0.7408 | 0.7392 | 0.7320 | 0.7200 |
| 0.8 | 0.6060 | 0.6340 | 0.6594 | 0.6816 | 0.7000 | 0.7139 | 0.7230 | 0.7269 | 0.7259 | 0.7200 | 0.7097 |

**Table 8: Sortino Ratio of *SPTR GCBC-ALOOP* (6/1/1988-12/31/2010)**

| $F_L \setminus F_S$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.7274 | 0.8056 | 0.8877 | 0.9705 | 1.0481 | 1.1125 | 1.1539 | 1.1652 | 1.1444 | 1.0971 | 1.0325 |
| 0.1 | 0.7628 | 0.8384 | 0.9082 | 0.9831 | 1.0498 | 1.1037 | 1.1381 | 1.1480 | 1.1320 | 1.0938 | 1.0397 |
| 0.2 | 0.7897 | 0.8591 | 0.9281 | 0.9938 | 1.0426 | 1.0902 | 1.1190 | 1.1278 | 1.1154 | 1.0844 | 1.0393 |
| 0.3 | 0.8095 | 0.8728 | 0.9345 | 0.9920 | 1.0421 | 1.0811 | 1.1018 | 1.1066 | 1.0970 | 1.0717 | 1.0340 |
| 0.4 | 0.8237 | 0.8812 | 0.9363 | 0.9869 | 1.0303 | 1.0639 | 1.0853 | 1.0930 | 1.0972 | 1.0574 | 1.0256 |
| 0.5 | 0.8334 | 0.8856 | 0.9349 | 0.9795 | 1.0175 | 1.0467 | 1.0654 | 1.0726 | 1.0681 | 1.0528 | 1.0410 |
| 0.6 | 0.8396 | 0.8869 | 0.9312 | 0.9708 | 1.0043 | 1.0299 | 1.0465 | 1.0532 | 1.0499 | 1.0373 | 1.0169 |
| 0.7 | 0.8430 | 0.8861 | 0.9259 | 0.9613 | 0.9910 | 1.0137 | 1.0285 | 1.0348 | 1.0324 | 1.0220 | 1.0046 |
| 0.8 | 0.8442 | 0.8835 | 0.9196 | 0.9514 | 0.9779 | 0.9982 | 1.0115 | 1.0174 | 1.0158 | 1.0071 | 0.9922 |

**Table 9: Stutzer Equivalent Sharpe Ratio of *SPTR GCBC-ALOOP* (6/1/1988-12/31/2010)**

| $F_L \setminus F_S$ | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5547 | 0.6031 | 0.6543 | 0.7061 | 0.7546 | 0.7946 | 0.8202 | 0.8275 | 0.8157 | 0.7877 | 0.7485 |
| 0.1 | 0.5797 | 0.6261 | 0.6690 | 0.7150 | 0.7558 | 0.7887 | 0.8097 | 0.8160 | 0.8072 | 0.7851 | 0.7531 |
| 0.2 | 0.5990 | 0.6410 | 0.6830 | 0.7228 | 0.7520 | 0.7802 | 0.7975 | 0.8030 | 0.7964 | 0.7790 | 0.7529 |
| 0.3 | 0.6137 | 0.6514 | 0.6883 | 0.7226 | 0.7523 | 0.7751 | 0.7871 | 0.7901 | 0.7852 | 0.7713 | 0.7500 |
| 0.4 | 0.6248 | 0.6586 | 0.6910 | 0.7207 | 0.7459 | 0.7653 | 0.7776 | 0.7821 | 0.7852 | 0.7631 | 0.7456 |
| 0.5 | 0.6332 | 0.6634 | 0.6919 | 0.7177 | 0.7394 | 0.7561 | 0.7667 | 0.7708 | 0.7686 | 0.7603 | 0.7546 |
| 0.6 | 0.6395 | 0.6665 | 0.6917 | 0.7143 | 0.7331 | 0.7475 | 0.7568 | 0.7607 | 0.7592 | 0.7526 | 0.7416 |
| 0.7 | 0.6443 | 0.6685 | 0.6909 | 0.7107 | 0.7272 | 0.7398 | 0.7480 | 0.7516 | 0.7507 | 0.7454 | 0.7362 |
| 0.8 | 0.6479 | 0.6697 | 0.6896 | 0.7071 | 0.7217 | 0.7328 | 0.7402 | 0.7436 | 0.7430 | 0.7388 | 0.7311 |

Here are some itemized findings from the back-tests and my comments:

1. Annualized returns increase monotonically with $F_L$ and $F_S$ except some minor exceptions for $F_S \geq 0.9$ when $F_L \leq 0.4$. *SPTR GCBC-ALOOP* ($F_L$=0.8 $F_S$=1) has the best annualized return of 17.17%. However, improvement in returns is much more sensitive to $F_L$ than to $F_S$. This

might be explained by the fact that signals from Golden Crosses have better chance to be "right" and are more profitable than those from Black Crosses (Table 2 and 3).

2. Given the range of $F_L$ and $F_S$, Stutzer Index changes very slightly the rankings of *SPTR GCBC-ALOOP* portfolios from those based on Sharpe Ratio or Sortino Ratio. *SPTR GCBC-ALOOP* ($F_L=0,\ F_S=0.7$) is about the portfolio that achieves the best risk-adjusted returns, which, however, are still less than those of *SPTR GC-PUT* in Table 4.
3. Fixing $F_L$, best risk-adjusted returns (Sharpe ratio, Sortino ratio or Stutzer Index) are achieved at $F_S \sim 0.7$. Fixing $F_S$, risk-adjusted returns increase with $F_L$ when $F_S < 0.2$, but decrease with $F_L$ when $F_S > 0.3$. At $F_S = 0.2 \sim 0.3$, risk adjusted returns are largely stable regardless of $F_L$. *SPTR GCBC-ALOOP* ($F_L=0.8,\ F_S=0.2$) has the best annualized return of *16.5%* when $F_S$ is fixed to *0.2*. It is among the limited range of *SPTR GCBC-ALOOP* cases that are feasible for implementation in a structured investment structure. An ETF benchmarked to *SPX* generally disallows holding more than 20 ~ 25% cash or cash equivalent which means $F_L \geq 0.75 \sim 0.8$ and $F_S \leq 0.2 \sim 0.25$. On the other hand, 20% *SPX* option margin requirement from Equation (2) sets $F_L \leq 0.8$. With excellent historical return and risk-adjusted return characteristics, *SPTR GCBC-ALOOP* ($F_L=0.8,\ F_S=0.2$) is an ideal candidate for an active ETF implementation. It also has no need to do sizable portfolio re-allocation between stock and cash on signal switching days as it satisfies $F_L + F_S = 1$. Nominally, it is a simple 80% stock index plus 20% cash passive investment portfolio, only actively changing the positions of written index options – shorting puts or calls monthly depending on a Golden Cross / Black Cross bullish or bearish trend prediction!
4. *SPTR GCBC-ALOOP* ($F_L=0.5,\ F_S=0.5$) is another nominal passive portfolio (a moderate 50% stock index / 50% cash allocation) with active overlay of written index options (Yang, 2010). *SPTR GCBC-ALOOP* ($F_L=0,\ F_S=1$) is holding 100% T-bills while actively writing *SPX* puts or calls to derive income – an option overlay cash management portfolio. Along with *SPTR GCBC-ALOOP* ($F_L=0.8,\ F_S=0.2$), these two special cases of *SPTR GCBC-ALOOP* have no exchange trades between stock index and T-bills beyond the portfolio rebalance as needed. For benchmark purpose, *SPTR GCBC-ALOOP* ($F_L=0,\ F_S=0$) is also examined, which turns out as simply switching between *BXM* and *PUT* indexes according to the Golden Cross and Black Cross signals. This implies 100% stock index / T-bills swap on signal triggering days.

Performance metrics and yearly returns during 06/01/1988 – 12/31/2010 for *SPTR GCBC-ALOOP* special cases are presented in Table 10 and 11, respectively. With the best annualized return from back-test, *SPTR GCBC-ALOOP* ($F_L=0.8\ F_S=1$) turn 1$ on 6/1/1988 into 35.92$ on

12/31/2010, barring any withdrawal, addition or taxes from the portfolio. In comparison, *SPTR* only turned a multiple of 7.803 for the period.

**Table 10: Performance Metrics of *SPTR GCBC-ALOOP* Cases (6/1/1988-12/31/2010)-**

| | SPTR GCBC-ALOOP ($F_L$= 0.5, $F_S$=0.5) | SPTR GCBC-ALOOP ($F_L$= 0, $F_S$=1) | SPTR GCBC-ALOOP ($F_L$= 0.8, $F_S$=1) | SPTR GCBC-ALOOP ($F_L$= 0.8, $F_S$=0.2) | SPTR GCBC-ALOOP ($F_L$= 0, $F_S$=0.7) | SPTR GCBC-ALOOP ($F_L$= 0, $F_S$=0) |
|---|---|---|---|---|---|---|
| **Annualized Return** | 14.88% | 11.49% | 17.17% | 16.48% | 11.45% | 10.66% |
| **Annualized Std Deviation** | 14.24% | 9.68% | 18.35% | 18.71% | 8.57% | 12.26% |
| **Sharpe Ratio** | 0.754 | 0.759 | 0.710 | 0.659 | 0.852 | 0.531 |
| **Sortino Ratio** | 1.077 | 1.064 | 1.018 | 0.941 | 1.208 | 0.737 |
| **Skew** | -0.627 | -0.723 | -0.501 | -0.504 | -0.761 | -0.407 |
| **Kurtosis** | 13.344 | 14.751 | 11.290 | 11.013 | 17.001 | 26.746 |
| **Alpha** | 7.67% | 6.97% | 10.59% | 7.65% | 6.07% | 3.29% |
| **Beta** | 0.570 | 0.070 | 0.454 | 0.873 | 0.229 | 0.600 |
| **Stutzer Index (Daily)** | 1.134E-03 | 1.112E-03 | 1.061E-03 | 9.435E-04 | 1.359E-03 | 6.105E-04 |
| **Stutzer Equivalent Sharpe Ratio** | 0.756 | 0.749 | 0.731 | 0.690 | 0.828 | 0.555 |
| **Maximum Drawdown** | 23.01% | 16.80% | 29.69% | 31.82% | 14.36% | 40.21% |
| **Ending Portfolio Value** | 22.975 | 11.689 | 35.920 | 31.432 | 11.586 | 9.865 |

**Table 11: Yearly Returns of *SPTR GCBC-ALOOP* Special Cases (6/1/1988-12/31/2010)**

| | SPTR GCBC-ALOOP ($F_L$= 0.5, $F_S$=0.5) | SPTR GCBC-ALOOP ($F_L$= 0, $F_S$=1) | SPTR GCBC-ALOOP ($F_L$= 0.8, $F_S$=1) | SPTR GCBC-ALOOP ($F_L$= 0.8, $F_S$=0.2) | SPTR GCBC-ALOOP ($F_L$= 0, $F_S$=0.7) | SPTR GCBC-ALOOP ($F_L$= 0, $F_S$=0) |
|---|---|---|---|---|---|---|
| **1988*** | 8.41% | 7.53% | 7.87% | 8.74% | 7.89% | 8.68% |
| **1989** | 37.00% | 24.58% | 44.89% | 44.89% | 24.58% | 24.58% |
| **1990** | -4.42% | 0.94% | -12.85% | -7.81% | 3.25% | 8.40% |
| **1991** | 23.56% | 11.30% | 22.77% | 31.58% | 14.13% | 20.87% |
| **1992** | 15.93% | 13.80% | 17.15% | 17.15% | 13.80% | 13.80% |
| **1993** | 17.90% | 14.14% | 20.18% | 20.18% | 14.14% | 14.14% |
| **1994** | 1.29% | 2.89% | -3.27% | 0.27% | 4.31% | 7.62% |
| **1995** | 33.19% | 16.88% | 43.93% | 43.93% | 16.88% | 16.88% |
| **1996** | 25.68% | 16.40% | 31.42% | 31.42% | 16.40% | 16.40% |
| **1997** | 43.54% | 27.68% | 53.63% | 53.63% | 27.68% | 27.68% |
| **1998** | 11.58% | 0.96% | 2.29% | 18.07% | 6.47% | 20.24% |
| **1999** | 29.50% | 20.98% | 34.39% | 34.39% | 20.98% | 20.98% |
| **2000** | 11.16% | 20.09% | 13.04% | 5.83% | 17.27% | 10.62% |
| **2001** | -3.50% | 2.96% | 2.50% | -7.80% | -0.94% | -10.92% |
| **2002** | 2.95% | 15.03% | 13.19% | -4.79% | 8.59% | -7.15% |
| **2003** | 27.24% | 12.07% | 28.38% | 36.77% | 15.04% | 21.55% |
| **2004** | 12.63% | 7.71% | 13.82% | 15.52% | 8.34% | 9.75% |
| **2005** | 7.40% | 6.71% | 7.67% | 7.67% | 6.71% | 6.71% |
| **2006** | 17.63% | 12.03% | 18.57% | 21.05% | 12.91% | 14.96% |
| **2007** | 10.55% | 10.32% | 11.12% | 10.49% | 10.08% | 9.54% |
| **2008** | -9.17% | 9.63% | 11.73% | -21.06% | -1.90% | -28.65% |
| **2009** | 37.73% | 20.20% | 42.52% | 47.87% | 22.87% | 26.72% |
| **2010** | -1.63% | -8.23% | -8.99% | 2.18% | -4.04% | 6.16% |

On risk measures, *SPTR GCBC-ALOOP* ($F_L$=0.8 $F_S$=1) has about the same standard deviation as the *SPTR,* but experienced a maximum drawdown within 30% - only about 55% that of the *SPTR.* All the special cases of *SPTR GCBC-ALOOP* in Table 10 delivered better annualized return than *SPTR* during the period of 22.6 years, with less maximum drawdown and better risk-adjusted return measures. From yearly returns of Table 5 and Table 11, however, *SPTR* out-performed all cases of *SPTR GCBC-ALOOP*, Golden Cross technical strategies and option writing indexes in the year 1998 and 2010. Interestingly, in the two preceding years, 1997 and 2009, *SPTR GCBC-ALOOP* ($F_L$=0.8, $F_S$=0.2) out-performed *SPTR* by over 20% in return each year. This will be further examined in section 5.

The monthly options roll-over, portfolio-rebalance (using the same methodology as *BXM* and *PUT* indexes), and market timing trades, can make transaction cost a potential performance drag for *SPTR GCBC-ALOOP*. For special cases of *SPTR GCBC-ALOOP* with leverage parameters $F_L + F_S$ =1, the market timing trades only involve swap written put and call options, and portfolio rebalance without any pre-defined exchange between equity index and T-bills. From 6/1/1988 to 12/31/2010, *271* times of portfolio rebalancing and option roll-over are needed at monthly option expiration Fridays and there were *20* market timing trades of swapping written puts and calls on Golden Cross/Black Cross days – a total of *291* times of trading in 22.6 years.

Assuming a cost of ***f*** basis points of the total portfolio value to cover the spreads in *SPX* options transactions and commissions for options trades and portfolio rebalancing, an ***f*** basis points loss is considered as the total market friction cost (excluding any tax, management fee and any potential market impact) of every one of *291* trades for any *SPTR GCBC-ALOOP* satisfying $F_L + F_S$ =1. For *f=5bp, 10bp, 15bp* and *20bp*[10], performance metrics of three special cases of *SPTR GCBC-ALOOP* are presented in Table 11(a-c), all for the time period 6/1/1988 to 12/31/2010.

Table 11(a) shows that with similar level of annualized return standard deviations, *SPTR GCBC-ALOOP* ($F_L$=0.8, $F_S$=0.2) has returned over 5% more per year on average than *SPTR* even charged with a market friction level of *f=10bp.* Despite a market friction load of *f=15bp*, *SPTR GCBC-ALOOP* ($F_L$=0, $F_S$=1) was able to delivered a similar annualized average return as *SPTR* with slightly over half of the return standard deviation of *SPTR*, as indicated in Table 11(b).

**Table 11 (a): Market Friction Effect on *SPTR GCBC-ALOOP* ($F_L$=0.8, $F_S$=0.2)**

| | ***SPTR*** | ***SPTR GCBC-ALOOP* ($F_L$ = 0.8, $F_S$ =0.2)** | | | | |
|---|---|---|---|---|---|---|
| | | ***f = 0 bp*** | ***f = 5 bp*** | ***f = 10 bp*** | ***f = 15 bp*** | ***f = 20 bp*** |
| **Annualized Return** | 9.52% | 16.48% | 15.74% | 15.00% | 14.26% | 13.53% |
| **Annualized Std Deviation** | 18.25% | 18.71% | 18.72% | 18.73% | 18.74% | 18.75% |
| **Sharpe Ratio** | 0.294 | 0.659 | 0.619 | 0.579 | 0.540 | 0.500 |
| **Sortino Ratio** | 0.421 | 0.941 | 0.882 | 0.824 | 0.767 | 0.710 |
| **Alpha** | 0.00 | 7.65% | 6.90% | 6.16% | 5.42% | 4.69% |
| **Beta** | 1.000 | 0.8727 | 0.8731 | 0.8734 | 0.8738 | 0.8742 |
| **Stutzer Equivalent Sharpe Ratio** | 0.367 | 0.690 | 0.655 | 0.621 | 0.587 | 0.552 |
| **Maximum Drawdown** | 55.25% | 31.82% | 32.40% | 32.97% | 33.53% | 34.09% |
| **Ending Portfolio Value** | 7.80 | 31.43 | 27.19 | 23.52 | 20.34 | 17.59 |

As shown in Table 11(c), even at largest estimated market friction of *$f$ = 20 bp,* the *SPTR GCBC-ALOOP* (*$F_L$=0.5, $F_S$=0.5*) out-performed *SPTR* and *SPTR GC-LEO* (performance metrics in Table 4) by 0.95% in annualized returns and matched *SPTR GC-LEO* in risk-adjusted returns. As an active benchmark portfolio, *SPTR GC-LEO* is of particular interest to compare to *SPTR GCBC-ALOOP* (*$F_L$=0.5, $F_S$=0.5*) based on *Delta* exposure, as to be shown in Section 5.

**Table 11 (b): Market Friction Effect on *SPTR GCBC-ALOOP* (*$F_L$=0, $F_S$=1*)**

| | ***SPTR*** | ***SPTR GCBC-ALOOP* (*$F_L$ = 0, $F_S$ =1*)** | | | | |
|---|---|---|---|---|---|---|
| | | ***f = 0 bp*** | ***f = 5 bp*** | ***f = 10 bp*** | ***f = 15 bp*** | ***f = 20 bp*** |
| **Annualized Return** | 9.52% | 11.49% | 10.78% | 10.07% | 9.36% | 8.66% |
| **Annualized Std Deviation** | 18.25% | 9.68% | 9.68% | 9.69% | 9.70% | 9.71% |
| **Sharpe Ratio** | 0.294 | 0.759 | 0.685 | 0.611 | 0.538 | 0.465 |
| **Sortino Ratio** | 0.421 | 1.064 | 0.958 | 0.853 | 0.748 | 0.645 |
| **Alpha** | 0.00 | 6.97% | 6.26% | 5.54% | 4.84% | 4.13% |
| **Beta** | 1.00 | 0.0696 | 0.0698 | 0.0700 | 0.0703 | 0.0705 |
| **Stutzer Equivalent Sharpe Ratio** | 0.367 | 0.749 | 0.683 | 0.616 | 0.550 | 0.484 |
| **Maximum Drawdown** | 55.25% | 16.80% | 17.13% | 17.47% | 17.80% | 18.13% |
| **Ending Portfolio Value** | 7.80 | 11.69 | 10.11 | 8.74 | 7.55 | 6.53 |

**Table 11 (c): Market Friction Effect on *SPTR GCBC-ALOOP* (*$F_L$=0.5, $F_S$=0.5*)**

| | ***SPTR*** | ***SPTR GCBC-ALOOP* (*$F_L$ = 0.5, $F_S$ =0.5*)** | | | | |
|---|---|---|---|---|---|---|
| | | ***f = 0 bp*** | ***f = 5 bp*** | ***f = 10 bp*** | ***f = 15 bp*** | ***f = 20 bp*** |
| **Annualized Return** | 9.52% | 14.88% | 14.14% | 13.41% | 12.68% | 11.96% |
| **Annualized Std Deviation** | 18.25% | 14.24% | 14.25% | 14.26% | 14.27% | 14.28% |
| **Sharpe Ratio** | 0.294 | 0.754 | 0.702 | 0.650 | 0.598 | 0.547 |
| **Sortino Ratio** | 0.421 | 1.077 | 1.001 | 0.925 | 0.851 | 0.777 |
| **Alpha** | 0.00 | 7.67% | 6.94% | 6.20% | 5.48% | 4.75% |
| **Beta** | 1.000 | 0.5695 | 0.5697 | 0.5700 | 0.5702 | 0.5704 |
| **Stutzer Equivalent Sharpe Ratio** | 0.367 | 0.756 | 0.711 | 0.666 | 0.621 | 0.576 |
| **Maximum Drawdown** | 55.25% | 23.01% | 23.09% | 23.17% | 23.25% | 23.33% |
| **Ending Portfolio Value** | 7.80 | 22.97 | 19.87 | 17.19 | 14.86 | 12.85 |

## 5. ***Delta*, Return Attribution and Risk Management for *SPTR GCBC-ALOOP***

The Greek Letter *Delta* of the *GCBC-ALOOP* can be obtained from equation (2) as:

$$\Delta = \frac{\partial \Pi}{\partial S} = \begin{cases} (1+F_L)-\delta, & When \quad 50DMA \geq 200DMA \quad on \quad S \\ (1-F_S)-\delta, & When \quad 50DMA < 200DMA \quad on \quad S \end{cases} \tag{6}$$

where $\delta$ is *Delta* of the European call option[11]. At monthly option expiration day or shortly after, portfolio *Delta* can be written as (see Appendix for details of the derivation, the volatility skew relationship and results on *SPX*):

$$\Delta = \frac{\partial \Pi}{\partial S} \approx \begin{cases} (\frac{1}{2} + F_L) - \frac{1}{\sqrt{2\pi}}\sigma_{BS}\sqrt{T} \cdot \dfrac{\partial \ln \sigma_{BS}}{\partial \ln S}, & when \quad S: 200DMA \le 50DMA \\ (\frac{1}{2} - F_S) - \frac{1}{\sqrt{2\pi}}\sigma_{BS}\sqrt{T} \cdot \dfrac{\partial \ln \sigma_{BS}}{\partial \ln S}, & when \quad S: 200DMA > 50DMA \end{cases} \tag{7}$$

When $F_L = F_S = 0$, equation (7) indicates standard covered buy-write (*CBW*) and collateralized put-write (*CPW*) have about the same $\Delta$. Calibrated with *SPX* and *VIX* data at monthly option expiration Fridays (see Appendix for details), $\Delta = 1 - \delta$ is estimated at 0.580 for the period from 1/19/1990 to 12/17/2010. This monthly *Delta* estimation is slightly smaller than the daily return beta *0.600* of *SPTR GCBC-ALOOP* ($F_L=0,\ F_S=0$) for the similar (but 1.5 year longer) period: 6/1/1988 to 12/31/2010 (Table 10). The deviations are mostly due to the slight out-of-money-ness of *SPX* options used in *BXM* and *PUT*, and the fact that their option components are rebalanced monthly rather than daily. It is also interesting to notice that this *Delta* estimation of *SPTR GCBC-ALOOP* ($F_L=0,\ F_S=0$) is between *PUT's* beta *0.571* and *BXM*'s beta *0.635* (see Table 4), and close to the lower *PUT's* beta. *SPTR GCBC-ALOOP* ($F_L=0,\ F_S=0$) spent more months following Golden Cross signal to take *PUT* position as a portfolio component than the months following Black Cross signal to take *BXM* position as a component.

Neglecting volatility skew effects represented by $\dfrac{\partial \ln \sigma_{BS}}{\partial \ln S}$ term in equation (7), $\Delta \times (SPTR - B) + B$ (using $\Delta$ as $\beta$ in CAPM formulae) defines an *active benchmark portfolio*:

$$\Pi^0_{SPTR}(F_L, F_S) = \begin{cases} \left(\frac{1}{2} + F_L\right) \cdot SPTR + \left(\frac{1}{2} - F_L\right) \cdot B, & when \quad 50DMA \ge 200DMA \quad on \quad SPTR \\ \left(\frac{1}{2} - F_S\right) \cdot SPTR + \left(\frac{1}{2} + F_S\right) \cdot B, & when \quad 50DMA < 200DMA \quad on \quad SPTR \end{cases} \tag{8}$$

In absence of volatility skew, it matches the *Delta* or beta exposure of *SPTR GCBC-ALOOP* with the same leverage factors $F_L$ and $F_S$. Excess return of $\Pi^0_{SPTR}(F_L, F_S)$ over that of *SPTR* represents an active alpha of the Golden Cross/Black Cross scheme of the active benchmark portfolio. It is obvious that $\Pi^0_{SPTR}(0.5, 0.5)$ is just *SPTR GC-LEO* from Equation (1). Another special case implies that when the volatility skew effect is omitted, *SPTR GCBC-ALOOP* ($F_L=0.8,\ F_S=0.2$) has 130% market exposure following Golden Cross signal and 30% market exposure following Black Cross signal. Taking the difference between *SPTR GCBC-ALOOP* (Equation 4) and the benchmark active portfolio $\Pi^0_{SPTR}(F_L, F_S)$ (Equation 8) leads to a self-financed, nominally *Delta* neutral active portfolio $V_{SPTR}$ that is independent of the leverage factors $F_L$ and $F_S$:

$$V_{SPTR} = \begin{cases} PUT - \frac{1}{2} \cdot (SPTR + B), & when \quad 50DMA \ge 200DMA \quad on \quad SPTR \\ BXM - \frac{1}{2} \cdot (SPTR + B), & when \quad 50DMA < 200DMA \quad on \quad SPTR \end{cases} \tag{9}$$

The return of $V_{SPTR}$ can define an "active *volatility skew premium*", as it corresponds to the market exposure of $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ term in equation (7). Using daily return data of *BXM, PUT* and *SPTR* Index, and the monthly option expiration Friday's rebalance, and the capital base of *BXM* or *PUT*, it is found the annualized return of $V_{SPTR}$ for the period 6/1/1988 – 12/31/2010 is *3.25*%.

Return for a *SPTR GCBC-ALOOP* is thus attributed to the return of its active benchmark portfolio, the active volatility skew premium, and their interaction. The component of interaction is calculated as any excess return of *SPTR GCBC-ALOOP* over the sum of returns of the active benchmark portfolio and the active volatility skew premium. For the return of the active benchmark portfolio $\Pi^0_{SPTR}(F_L, F_S)$, it can be further attributed to an active alpha over *SPTR*. *SPTR* also has a commonly defined *Equity Risk Premium* over risk free rate. Table 12 listed the return attribution for the three special cases of *SPTR GCBC-ALOOP* mentioned earlier that satisfy $F_L + F_S = 1$. The interaction turns out to be always positive for the whole period.

**Table 12: Return Attribution of *SPTR GCBC-ALOOP* Cases***

**(6/1/1988 – 12/31/2010)**

| | **SPTR GCBC-ALOOP ($F_L$= 0.8, $F_S$=0.2)** | **SPTR GCBC-ALOOP ($F_L$= 0.5, $F_S$=0.5)** | **SPTR GCBC-ALOOP ($F_L$= 0, $F_S$=1)** |
|---|---|---|---|
| **Risk Free Rate** | 4.15% | 4.15% | 4.15% |
| **Equity Risk Premium** | 5.37% | 5.37% | 0.00% |
| **Active Benchmark Alpha** | 3.12% | 1.49% | 3.64% |
| **Volatility Skew Risk Premium** | 3.25% | 3.25% | 3.25% |
| **Interaction of Active Alpha/ Volatility Skew Premium** | 0.59% | 0.62% | 0.45% |
| **Total Return** | **16.48%** | **14.87%** | **11.49%** |

*In computing *active benchmark alpha* (alpha of active benchmark portfolio), *SPTR GCBC-ALOOP* (*$F_L$=0.8, $F_S$=0.2*) and *SPTR GCBC-ALOOP* (*$F_L$=0.5, $F_S$=0.5*) use *SPTR* as the benchmark, while *SPTR GCBC-ALOOP* (*$F_L$=0, $F_S$=1*) uses 3-month T-Bills as the benchmark.

As the slope of volatility skew represented by $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ is mostly negative[12] (estimated from historical *VIX* and *SPX* data in the Appendix), market exposure from both the active benchmark portfolio and volatility skew term in equation (7) are expected to be positive in bullish period following a Golden Cross. Return from their interaction could also be positive, because *SPTR* index historically had gains much more often than losses in the months following Golden Cross (Table 3). When *SPX* changed month-to-month at least 1%, the months in the bullish Golden Cross period outnumbered Black Cross period at 144 to 54 (Appendix Table A4). In a

Black Cross period, slope of volatility skew $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ was also more often negative (47 out of 54 months from Appendix Table A4). This also leads to an increased market exposure to *SPTR*.

**Table 13 (a): *SPTR GCBC-ALOOP* ($F_L$=0.5, $F_S$=0.5) Return Attribution Year by Year (1988-2010)**

| Year | Total Alpha | Active Benchmark Alpha | Volatility Skew Risk Premium | Interaction |
|---|---|---|---|---|
| 1988* | 2.08% | -1.05% | 1.80% | 1.33% |
| 1989 | 5.32% | 0.00% | 4.51% | 0.80% |
| 1990 | -1.31% | -6.31% | 5.47% | -0.48% |
| 1991 | -6.91% | -10.20% | 2.22% | 1.07% |
| 1992 | 8.31% | 0.00% | 6.94% | 1.38% |
| 1993 | 7.82% | 0.00% | 7.11% | 0.71% |
| 1994 | -0.03% | -4.37% | 4.51% | -0.16% |
| 1995 | -4.39% | 0.00% | -3.00% | -1.39% |
| 1996 | 2.72% | 0.00% | 0.92% | 1.81% |
| 1997 | 10.17% | 0.00% | 7.93% | 2.25% |
| 1998 | -17.00% | -20.26% | 2.75% | 0.51% |
| 1999 | 8.72% | 0.00% | 7.57% | 1.15% |
| 2000 | 20.27% | 8.07% | 11.21% | 0.99% |
| 2001 | 8.39% | 15.50% | -6.45% | -0.65% |
| 2002 | 25.05% | 20.77% | 4.93% | -0.65% |
| 2003 | -1.44% | -9.16% | 6.17% | 1.55% |
| 2004 | 1.75% | -1.96% | 3.07% | 0.64% |
| 2005 | 2.49% | 0.00% | 2.20% | 0.29% |
| 2006 | 1.84% | -2.89% | 4.14% | 0.59% |
| 2007 | 5.06% | 0.76% | 3.66% | 0.64% |
| 2008 | 27.83% | 38.43% | -10.31% | -0.29% |
| 2009 | 11.27% | -3.57% | 10.34% | 4.50% |
| 2010 | -16.69% | -15.19% | -1.01% | -0.48% |

*1988 is recorded for partial year (6/1/1988 to 12/31/1988). "Total Alpha" is the difference of annual returns between *SPTR GCBC-ALOOP* ($F_L$=0.5, $F_S$=0.5) and *SPTR*. "Active Benchmark" refers to *SPTR GC-LEO* and "Active Benchmark Alpha" is the difference of annual returns between *SPTR GCBC-LEO* and *SPTR.*

When examining *SPTR GCBC-ALOOP* return attribution in each year (Table 13 a-c) for 1988-2010, it is found that "active benchmark return" dominates the level of overall alpha and interaction return contributions were smaller. Only four years had negative volatility skew premium (1995, 2001, 2008 and 2010). The worst annual "active benchmark alpha" happened in 1998 and 2010, that led to the largest annual under-performance of *SPTR GCBC-ALOOP* ($F_L$=0.8, $F_S$=0.2), over 10% below *SPTR* in both years (see Table 13(b)). For the preceding two years (1997 and 2009), *SPTR GCBC-ALOOP* ($F_L$=0.8, $F_S$=0.2) had the best total alphas of over 20% due to superior volatility skew premium and interaction returns during those two years.

For monthly performance between option expiration Fridays, volatility skew can interact with a Golden Cross or Black Cross signal to exacerbate underperformance of *SPTR GCBC-ALOOP* ($F_L$*=0.5,* $F_S$*=0.5*) relative to its active benchmark portfolio *SPTR GC–LEO*. For example, on 8/21/1998, *VIX* closed at $\sigma_{BS} = 33.14\%$, and *ex post* estimate of volatility skew $\frac{\partial \ln \sigma_{BS}}{\partial \ln S} = -7.66$. This caused the effective $\Delta \sim 1.29$ for *SPTR GCBC-ALOOP* ($F_L$*=0.5,* $F_S$*=0.5*): an extra 29% market exposure when the following month was still in Golden Cross bullish period but *SPTR* declined. On 9/19/2008, monthly *ex post* estimate $\frac{\partial \ln \sigma_{BS}}{\partial \ln S} = -14.61$ with *VIX* at $\sigma_{BS} = 32.07\%$ to cause the effective $\Delta \sim 0.54$. Thus the volatility skew effect led to an extra 54% portfolio market exposure even when the period was bearish under a correct Black Cross signal.

**Table 13 (b): *SPTR GCBC-ALOOP* (*$F_L$=0.8, $F_S$=0.2*) Return Attribution**

**Year by Year (1988-2010)**

| Year | Total Alpha | Active Benchmark Alpha | Volatility Skew Risk Premium | Interaction |
|---|---|---|---|---|
| 1988* | 2.41% | -0.68% | 1.80% | 1.28% |
| 1989 | 13.20% | 7.26% | 4.51% | 1.42% |
| 1990 | -4.70% | -9.40% | 5.47% | -0.78% |
| 1991 | 1.11% | -2.75% | 2.22% | 1.64% |
| 1992 | 9.53% | 1.02% | 6.94% | 1.57% |
| 1993 | 10.10% | 2.00% | 7.11% | 0.99% |
| 1994 | -1.05% | -5.39% | 4.51% | -0.18% |
| 1995 | 6.35% | 11.06% | -3.00% | -1.70% |
| 1996 | 8.46% | 5.46% | 0.92% | 2.08% |
| 1997 | 20.27% | 8.82% | 7.93% | 3.52% |
| 1998 | -10.51% | -14.19% | 2.75% | 0.93% |
| 1999 | 13.62% | 4.39% | 7.57% | 1.65% |
| 2000 | 14.94% | 2.88% | 11.21% | 0.85% |
| 2001 | 4.09% | 11.06% | -6.45% | -0.51% |
| 2002 | 17.31% | 13.72% | 4.93% | -1.35% |
| 2003 | 8.09% | -0.22% | 6.17% | 2.14% |
| 2004 | 4.64% | 0.78% | 3.07% | 0.78% |
| 2005 | 2.76% | 0.27% | 2.20% | 0.29% |
| 2006 | 5.25% | 0.26% | 4.14% | 0.85% |
| 2007 | 5.00% | 0.48% | 3.66% | 0.85% |
| 2008 | 15.94% | 26.50% | -10.31% | -0.26% |
| 2009 | 21.40% | 5.71% | 10.34% | 5.35% |
| 2010 | -12.88% | -11.33% | -1.01% | -0.54% |

There are eight scenarios with the combinations of a "right" or "wrong" Golden Cross or Black Cross signal, and a negative or positive volatility skew $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$. Two of them stand out from a risk management perspective for a *SPTR GCBC-ALOOP* portfolio:

**Table 13 (c): *SPTR GCBC-ALOOP* ($F_L$=0, $F_S$=1) Return Attribution Year by Year (1988-2010)**

| Year | Total Alpha | Active Benchmark Alpha | Volatility Skew Risk Premium | Interaction |
|---|---|---|---|---|
| 1988* | 3.03% | -0.06% | 1.80% | 1.28% |
| 1989 | 15.72% | 11.13% | 4.51% | 0.08% |
| 1990 | -7.27% | -12.72% | 5.47% | -0.02% |
| 1991 | 5.53% | 2.58% | 2.22% | 0.73% |
| 1992 | 10.16% | 2.10% | 6.94% | 1.13% |
| 1993 | 10.98% | 3.50% | 7.11% | 0.36% |
| 1994 | -1.64% | -6.06% | 4.51% | -0.09% |
| 1995 | 10.98% | 14.90% | -3.00% | -0.92% |
| 1996 | 11.00% | 8.64% | 0.92% | 1.44% |
| 1997 | 22.26% | 13.64% | 7.93% | 0.69% |
| 1998 | -4.15% | -7.01% | 2.75% | 0.11% |
| 1999 | 15.99% | 8.08% | 7.57% | 0.34% |
| 2000 | 13.83% | 0.95% | 11.21% | 1.67% |
| 2001 | -0.65% | 6.83% | -6.45% | -1.03% |
| 2002 | 13.36% | 8.46% | 4.93% | -0.03% |
| 2003 | 11.02% | 4.32% | 6.17% | 0.53% |
| 2004 | 6.27% | 2.79% | 3.07% | 0.41% |
| 2005 | 3.40% | 0.93% | 2.20% | 0.27% |
| 2006 | 7.01% | 2.60% | 4.14% | 0.27% |
| 2007 | 5.64% | 1.50% | 3.66% | 0.48% |
| 2008 | 8.19% | 19.38% | -10.31% | -0.88% |
| 2009 | 20.05% | 7.07% | 10.34% | 2.64% |
| 2010 | -8.37% | -7.01% | -1.01% | -0.34% |

1. A negative volatility skew $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ under a wrong Golden Cross signal. This could be a situation that market declines quickly after a bull-run and market fear gage *VIX* spikes to the upside.
2. A positive volatility skew $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ under a wrong Black Cross signal. This could be a situation that the market rise from lows but *VIX* still increases, as market participants disbelieve the bounce and buy further into *SPX* options for protection.

To address the issue of *SPTR GCBC-ALOOP* portfolio risk management, two trading rules within an *SPX* option month are proposed as an example for back testing[13]. Aiming to correct the wrong market exposure as expressed in equation (7), they are named as *Delta trades*:

1. In the bullish period following a *SPTR* Golden Cross, on the *first* Friday after last option expiration, check the following two criteria at close to 4:00PM:

$$\ln\left(SPX_{t+n}/SPX_t\right) < -1\%\text{, and } VIX_{t+n} \times \left(\frac{\ln VIX_{t+n} - \ln VIX_t}{\ln SPX_{t+n} - \ln SPX_t}\right) < -2$$

Where $t$ denotes the close prices of last option expiration Friday and $n$=1 week. Only when both are satisfied, trade to the bearish branch position (Equation 4) for the *SPTR GCBC-ALOOP* portfolio on the same day, and keep the portfolio position until next monthly option expiration.

2. In the bearish period following a *SPTR* Black Cross, on the *first* or *second* Friday after last option expiration, check the following two criteria at close to 4:00PM:

$$\ln\left(SPX_{t+n}/SPX_t\right) > \tfrac{1}{4}\%\text{, and } VIX_{t+n} \times \left(\frac{\ln VIX_{t+n} - \ln VIX_t}{\ln SPX_{t+n} - \ln SPX_t}\right) > 1$$

Where $t$ denotes the close prices of last option expiration Friday and $n$=1 week or 2 weeks. Only when both are satisfied, trade to the bullish branch position for the *SPTR GCBC-ALOOP* portfolio (Equation 4) on the same day, and keep the portfolio position until next monthly option expiration.

Both types of *Delta* trades check weekly the adverse portfolio market exposure early in an option month. The criteria are derived from Equation (7) with threshold values based on observation of statistical outlier of volatility skew (see Appendix). The *Delta trades* switch the direction of portfolio leverage assuming the unfavorable market rising or declining trend since last option expiration continues until the next option expiration.

**Table 14 (a): Performance of *SPTR GCBC-ALOOP* (*$F_L$=0.8, $F_S$=0.2*) *Delta* Trades During 1/19/1990 to 12/31/2010**

| Option Month (ending) | *SPTR* Monthly Return | *Delta* Trade Date | Direction of *Delta* Trade | Monthly Return of SPTR GCBC-ALOOP with *Delta* Trade | Monthly Return of SPTR GCBC-ALOOP without *Delta* Trade | *Delta* Trade Net Monthly P/L |
|---|---|---|---|---|---|---|
| 8/17/1990 | -8.99% | 7/27/1990 | Put-Write to Call-Write | -7.79% | -14.61% | 6.81% |
| 12/21/1990 | 4.90% | 11/30/1990 | Call-Write to Put-Write | 4.24% | 2.05% | 2.19% |
| 8/21/1998 | -8.75% | 7/24/1998 | Put-Write to Call-Write | -9.30% | -13.45% | 4.14% |
| 10/20/2000 | -4.62% | 9/22/2000 | Put-Write to Call-Write | -2.87% | -7.32% | 4.45% |
| 5/18/2001 | 4.08% | 4/27/2001 | Call-Write to Put-Write | 4.74% | 2.37% | 2.38% |
| 8/16/2002 | 9.73% | 8/2/2002 | Call-Write to Put-Write | 10.11% | 4.28% | 5.83% |
| 2/15/2008 | 2.05% | 1/25/2008 | Call-Write to Put-Write | 5.09% | 3.28% | 1.81% |
| 5/15/2009 | 1.75% | 5/1/2009 | Call-Write to Put-Write | 3.14% | 2.24% | 0.90% |
| 2/19/2010 | -2.15% | 1/21/2010* | Put-Write to Call-Write | -1.69% | -2.69% | 0.99% |
| | | 1/22/2010 | | -3.90% | | -1.21% |
| 6/18/2010 | -1.36% | 5/19/2010* | Put-Write to Call-Write | -2.65% | -4.96% | 2.31% |
| | | 5/21/2010 | | -8.33% | | -3.37% |

*The two *Delta* trades in 2010 can result in net gains if the criteria are checked daily during the first week of the option month rather than waiting until first Friday. As a result, the January 2010 *Delta* trade was pulled one day ahead to 1/21/2010, and the May 2010 *Delta* trade was pulled two days ahead to 5/19/2010.

### Table 14 (b): Performance of *SPTR GCBC-ALOOP* ($F_L$*=0.5,* $F_S$*=0.5*) *Delta* Trades During 1/19/1990 to 12/31/2010

| Option Month (ending) | *SPTR* Monthly Return | *Delta* Trade Date | Direction of *Delta* Trade | Monthly Return of SPTR GCBC-ALOOP with *Delta* Trade | Monthly Return of SPTR GCBC-ALOOP without *Delta* Trade | *Delta* Trade Net Monthly P/L |
|---|---|---|---|---|---|---|
| 8/17/1990 | -8.99% | 7/27/1990 | Put-Write to Call-Write | -5.00% | -11.78% | 6.78% |
| 12/21/1990 | 4.90% | 11/30/1990 | Call-Write to Put-Write | 2.91% | 0.72% | 2.18% |
| 8/21/1998 | -8.75% | 7/24/1998 | Put-Write to Call-Write | -6.63% | -10.72% | 4.09% |
| 10/20/2000 | -4.62% | 9/22/2000 | Put-Write to Call-Write | -1.36% | -5.80% | 4.44% |
| 5/18/2001 | 4.08% | 4/27/2001 | Call-Write to Put-Write | 3.58% | 1.20% | 2.37% |
| 8/16/2002 | 9.73% | 8/2/2002 | Call-Write to Put-Write | 7.20% | 1.39% | 5.81% |
| 2/15/2008 | 2.05% | 1/25/2008 | Call-Write to Put-Write | 4.51% | 2.71% | 1.80% |
| 5/15/2009 | 1.75% | 5/1/2009 | Call-Write to Put-Write | 2.62% | 1.72% | 0.89% |
| 2/19/2010 | -2.15% | 1/21/2010* | Put-Write to Call-Write | -1.04% | -2.04% | 1.00% |
| | | 1/22/2010 | | -3.22% | | -1.18% |
| 6/18/2010 | -1.36% | 5/19/2010* | Put-Write to Call-Write | -2.23% | -4.55% | 2.32% |
| | | 5/21/2010 | | -7.84% | | -3.29% |

### Table 14 (c): Performance of *SPTR GCBC-ALOOP* ($F_L$*=0,* $F_S$*=1*) *Delta* Trades During 1/19/1990 to 12/31/2010

| Option Month (ending) | *SPTR* Monthly Return | *Delta* Trade Date | Direction of *Delta* Trade | Monthly Return of SPTR GCBC-ALOOP with *Delta* Trade | Monthly Return of SPTR GCBC-ALOOP without *Delta* Trade | *Delta* Trade Net Monthly P/L |
|---|---|---|---|---|---|---|
| 8/17/1990 | -8.99% | 7/27/1990 | Put-Write to Call-Write | -0.29% | -7.07% | 6.79% |
| 12/21/1990 | 4.90% | 11/30/1990 | Call-Write to Put-Write | 0.71% | -1.49% | 2.20% |
| 8/21/1998 | -8.75% | 7/24/1998 | Put-Write to Call-Write | -2.09% | -6.16% | 4.07% |
| 10/20/2000 | -4.62% | 9/22/2000 | Put-Write to Call-Write | 1.18% | -3.28% | 4.45% |
| 5/18/2001 | 4.08% | 4/27/2001 | Call-Write to Put-Write | 1.64% | -0.73% | 2.37% |
| 8/16/2002 | 9.73% | 8/2/2002 | Call-Write to Put-Write | 2.40% | -3.43% | 5.82% |
| 2/15/2008 | 2.05% | 1/25/2008 | Call-Write to Put-Write | 3.54% | 1.74% | 1.80% |
| 5/15/2009 | 1.75% | 5/1/2009 | Call-Write to Put-Write | 1.74% | 0.86% | 0.89% |
| 2/19/2010 | -2.15% | 1/21/2010* | Put-Write to Call-Write | 0.04% | -0.96% | 1.00% |
| | | 1/22/2010 | | -2.13% | | -1.16% |
| 6/18/2010 | -1.36% | 5/19/2010* | Put-Write to Call-Write | -1.54% | -3.87% | 2.32% |
| | | 5/21/2010 | | -7.07% | | -3.20% |

Table 14(a-c) lists all the ten *Delta* trades in the whole back test period and their performance for three special cases of *SPTR GCBC-ALOOP* with $F_L + F_S = 1$. All five *Delta* trades during bearish periods following a Black Cross result in net gains compared to the original active portfolio without *Delta* trades. Three pre-2010 *Delta* trades during bullish periods following a Golden Cross result in relative larger net gains (all over +4%). Two *Delta* trades in

2010 are both bullish period risk management type that ended in modest monthly net loss. However daily monitoring to trigger the couple of *Delta* trades one or two days earlier can lead to net monthly gains. Since leverage parameters have $F_L + F_S = 1$ for the *SPTR GCBC-ALOOP* cases, the *Delta* trades only involve closing written call options in *BXM* and then short put options in *PUT* (call-write to put-write type) in the bearish Black Cross period, or closing written put options and then short call options (put-write to call-write type) in the bullish Golden Cross period. The amount of rebalance between underlying *SPX* index position and the T-bills is negligible for *Delta* trades. Thus the net gains in the last columns of Table 14(a-c) are essentially the same for the same date of *Delta* trades regardless different leverage factors for the three Table's *SPTR GCBC-ALOOP* cases.

## 6. <u>Concluding Remarks</u>

For over two decades, the average realized volatilities for S&P 500 index has been below the average implied volatilities for *SPX* index options. This is often quoted as the direct reason to pursue out-performance through index option writing strategies. However, passive buy-write or put-write portfolio strategies may not be fully effective to take advantage of the average negative volatility premium embedded in index options[14].

As identified by Hill *et al* (2006), exercise cost (payment to settle in-the-money written options at expiration) turned out as the largest performance drag on passive buy-write strategies. In a rising market, buy-write strategy under-performs the underlying equity index due to non-zero exercise cost even when the market gains less than what was initially implied by the ATM index option premium. When market declines, a buy-write strategy like *BXM* avoids exercise cost but the portfolio still ends with a loss, since the call premium collected is not enough to offset the loss from fully invested underlying index position (as call $|Delta| < 1$). A collateralized put-write strategy like *PUT* index will incur exercise cost when market declines. The portfolio can also suffer disastrous losses under market stress, e.g. *PUT* index has a daily paper loss of 24.4% on 10/19/1987, worse than S&P 500 index! Even when market rises, large cash positions required by put-write strategy make funding expense and opportunity cost concerns for investors, and also prevent its implementation in form of a structured product investment.

The long term persistence of rich index option premium since 1987 market crash led to more fundamental, market structural, and behavioral finance based explanations. Given the historical regulatory and technology limitations, lack of common, liquid and effective investment vehicles to arbitrage away the negative "realized – implied volatility spread" could be added to the long list of interpretations. The current approach of *ALOOP* (Active Leveraged Option Overly Portfolio) addresses the directional deficiencies of standard buy-write and put-write strategies by trying to avoid all exercise costs. It relies on reasonable predictability of mid-to-long term trends of stock market index daily time series. For example, Golden Cross / Black Cross signals are found effective from an empirical analysis of historical data; further, the introduction of directional leverage for the portfolio agrees with the theoretical framework of Zhu and Zhou (2009) to switch fixed portfolio weights according to a moving average technical analysis rule.

From the portfolio *Delta* expression of Equation (7), a negative slope of volatility skew can be understood as an *ex ante* representation of alpha source of the negative "realized – implied volatility spread". An overall positive return from the interaction between volatility skew risk premium and active benchmark alpha is found from the 22.6 years of back test, indicating the portfolio mechanism of combining active leverage and option overlay as value adding. By adjusting the level of active leverage, specific portfolio can be designed to suit to aggressive, moderate or conservative investors. Short term index option trades are back-tested against the problematic market timing signals, and they turned out to be effective for portfolio risk management under adverse market environment.

Although S&P 500 Index is chosen as the underlying index in this study, the current approach may be applied to other broad market indexes such as Dow Jones Industrial Average, NASDAQ 100 and Russell 2000, and compare to their respective existing option strategy benchmark index (Buy-Write) from *CBOE*. The basic premise of the portfolio *Delta* estimate and *Delta* trades is for liquid, continuous equity indexes without price jumps like in individual stocks.

Some special cases of the *ALOOP* portfolios, e.g., *SPTR-GCBC ALOOP* ($F_L=0.8$, $F_S=0.2$), can be implemented as an alternative to common equity index investments. Given the potential market size, market impact of extra option transactions could be important. On dates when option contracts are rolled over or a technical signal is triggered, additional selling supply of index options can press the bid prices lower. The directional leverage implied in these option positions can cause additional momentum for the underlying index at these key dates. However, a diversification of rebalance schedules and technical signals[15] used among market participants could mitigate potential trading event risk. At other times, however, the convex nature of index option writing can dampen market move and possibly reduce underlying index volatility gradually. The mechanism can be that option market maker or other counter parties *Delta* hedge their positions in the underlying or index futures markets. Lower realized index volatility reduces average exercise cost and compensates potential lower option premium collected by an index writing portfolio. In that sense, aligned with a liquid and efficient underlying equity index market, an active index writing portfolio can expect its efficacy, as observed in the 22.6 years of back test, to last for decades to come.

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## Appendix: Estimation of Greek Letter *Delta* under Volatility Skew

Under Black-Scholes Option Pricing Theory, the *Delta* of a European call option (Hull, 2004) is: $\delta_{BS} = \exp[-q \cdot T] \cdot N(d_1)$ where $q$ denotes the continuous dividend yield, $T$ is the time to expiration, $N(d_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d_1} \exp(-x^2/2)dx$ and $d_1 = [\ln(S/K) + (r_f - q)] \big/ (\sigma_{BS}\sqrt{T}) + \sigma_{BS}\sqrt{T}/2$ in which $K$ is the strike price and $\sigma_{BS}$ is the Black-Scholes implied volatility.

The call option *Delta* can be written as: $\delta = \delta_{BS} + \frac{\partial C}{\partial \sigma_{BS}} \cdot \frac{\partial \sigma_{BS}}{\partial S}$ where the $\frac{\partial \sigma_{BS}}{\partial S} \approx \frac{\partial \sigma_{BS}}{\partial K}$ (Derman *et al* 1996) represents the volatility skew and $\frac{\partial C}{\partial \sigma_{BS}} = \nu = \frac{1}{\sqrt{2\pi}} S\sqrt{T} \exp[-d_1^2/2 - q \cdot T]$

For the case of at the money call $K = S$, thus $d_1$ is reduced to:

$$\overline{d}_1 = (r_f - q)/(\sigma_{BS}\sqrt{T}) + \sigma_{BS}\sqrt{T}/2 \tag{A.0}$$

Look at the short maturity option *Delta* at about one month before expiration: $T \sim \frac{1}{12}$ year:

$$\delta = A_0 + A_1 \cdot \left(\sigma_{BS}\sqrt{T}\right) \cdot \left(\frac{\partial \ln \sigma_{BS}}{\partial \ln S}\right) \tag{A.1}$$

where $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ is directly related to the volatility skew level and two coefficients $A_1$ and $A_0$ are defined as:

$$A_0 = \frac{\exp(-q \cdot T)}{\sqrt{2\pi}} \int_{-\infty}^{\overline{d}_1} \exp\left(-\frac{x^2}{2}\right) \cdot dx = \exp(-q \cdot T) \cdot erfc\left(\overline{d}_1\right) \tag{A.2}$$

$$A_1 = \frac{1}{\sqrt{2\pi}} \exp\left(-q \cdot T - \overline{d}_1^2 \big/ 2\right) \tag{A.3}$$

For *SPX* call options, $\sigma_{BS} \approx 20.0\%$ from the arithmetic average of *VIX* daily close of option expiration Fridays from 1/19/1990 to 12/17/2010, is taken as the surrogate of implied volatility $\sigma_{BS}$. Other parameters can be estimated for the period as: $r_f = 3.82\%$ using annualized 3-month T-bill return; $q = 2.30\%$ (calculated through the difference between annualized returns of *SPTR* and *SPX*). However, with the expression of $\overline{d}_1$ in Equation (A.0) changed signs multiple times due to the relationship between risk free rate $r_f$ and on-going *SPX* dividend yield $q$ (e.g. difference between one year trailing returns of *SPTR* and *SPX*) in the past 21 years, a neutral estimate of $\overline{d}_1$ is taken as zero that leads to: $A_0 \approx \frac{1}{2}$ and $A_1 \approx \frac{1}{\sqrt{2\pi}}$. Equation (A.1) is simplified as:

$$\delta \approx \frac{1}{2} - \frac{1}{\sqrt{2\pi}} \cdot \left(\sigma_{BS}\sqrt{T}\right) \cdot \left(\frac{\partial \ln \sigma_{BS}}{\partial \ln S}\right) \tag{A.4}$$

Thus, $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ as a representation of volatility skew becomes critical to determine *Delta* for at the money call options. Absence of volatility skew (i.e. when the implied volatility is independent of underlying equity index or strike price), the nominal *Delta* of the short term (about one month to maturity) call or put option is about 0.5. I use *VIX* and *SPX* index daily close values on option expiration Fridays every month to estimate its value as: $\left(\frac{\Delta \ln \sigma_{BS}}{\Delta \ln S}\right)_n = \frac{\ln VIX_{n+1} - \ln VIX_n}{\ln S_{n+1} - \ln S_n}$ where the subscripts *n* and *n+1* denote consecutive monthly option expiration Friday close values.

Excluding the months that month-to-month *SPX* change is less than 1% which are considered as singularity situations for $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$, the history and distribution of 198 monthly discrete $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ estimations are shown in Figure A1 and A2, with a mean of *-3.45*. Thus using equation (A.4), $\delta \sim 0.420$ for the 21 year period (1/19/1990 – 12/17/2010).

Historical monthly estimations of $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ for *SPX* for the same period are shown in Figure A3. $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S} < -2$ and $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S} > 1$ appear as the statistical outliers (two levels marked as dotted lines in Figure A3). Considering equation A.4, estimated value of $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ can be used as risk control threshold to avoid tail event losses in index option writing strategies such as *PUT* and *BXM*. Both *PUT* and *BXM* have the same nominal equity index exposure of $\frac{1}{2} - \sqrt{\frac{T}{2\pi}} \cdot \left(\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S}\right)$, despite that both could be slightly out-of-money and $\delta_{BXM} > \delta_{PUT}$ (because $d_1 > \overline{d}_1$ for *PUT* and $d_1 < \overline{d}_1$ for *BXM*).

Further separating the 198 monthly estimations of $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ and $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ according to *SPTR* market timing periods of Golden Cross/Black Cross and their positive/negative value, the average values are listed for various groupings in Table A4.

When $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S} \le -2$ at a monthly option expiring Friday close, nominal *Delta* of a written *SPX* put option (e.g. CBOE's *PUT* Index) increases from *0.5* to *0.73* (estimated by equation A.4 with $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S} = -2$). This is an unfavorable change in market exposure when

*SPX* is declining. On the other hand, when *SPX* is rising and $\sigma_{BS}\frac{\partial \ln \sigma_{BS}}{\partial \ln S} > 1$ at an option expiring Friday close, nominal *Delta* of a *SPX* covered call strategy (e.g. the CBOE *BXM* index) decreases from *0.5* to *0.385* (estimated by equation A.4 with $\sigma_{BS}\frac{\partial \ln \sigma_{BS}}{\partial \ln S} = 1$) – an unfavorable change in market exposure if underlying index moves to the upside.

**Figure A1: Monthly $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ in 1990-2010**
**(Monthly Change in *VIX* divided by Monthly Change in *SPX*)**

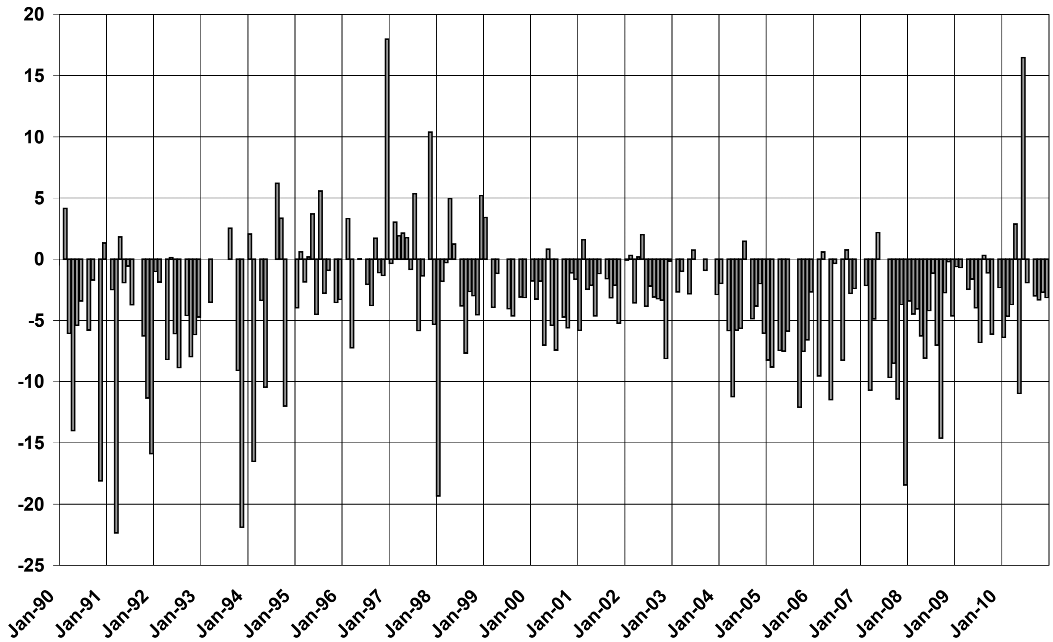

**Figure A2: Histogram of Monthly $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ in 1990-2010 (Mean = *-3.45*)**

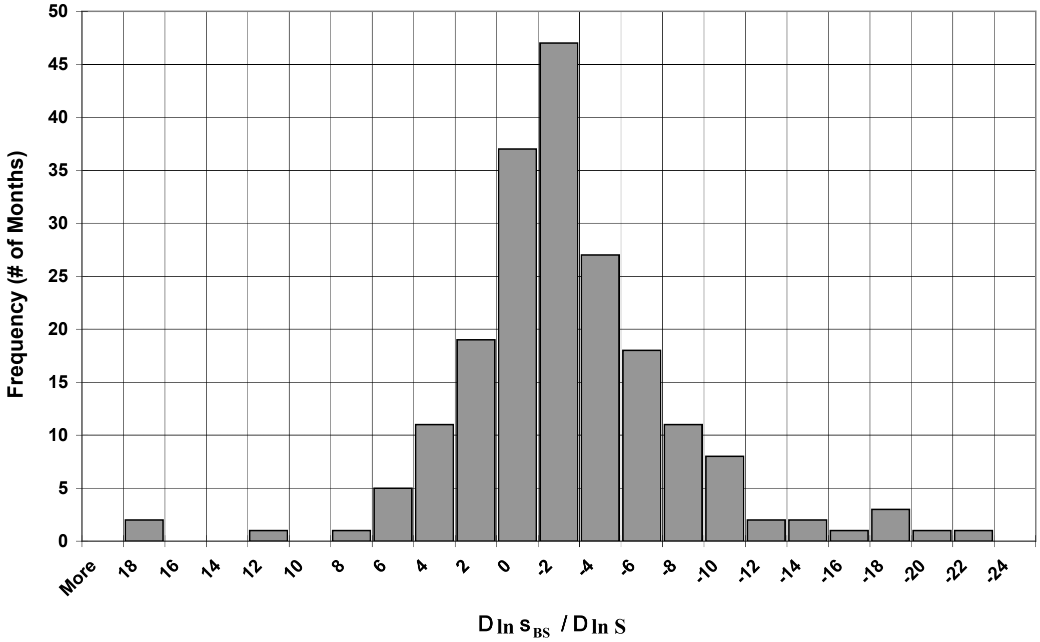


**Figure A3:** **Monthly $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ in 1990-2010:**

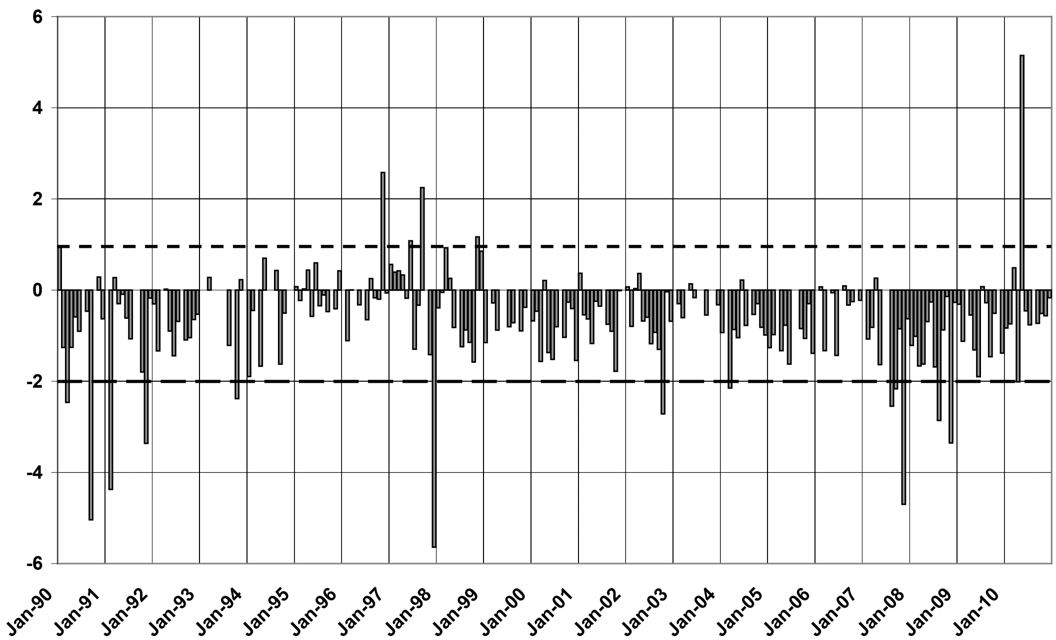

**Table A4: Golden Cross/Black Cross, Positive or Negative Volatility Skew and Average Monthly Values from 1/19/1990 to 12/17/2010***

| Number of Months (SPX change > 1%) | Number of Months Following Signal of | Number of Months with Volatility Skew | Average $\frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ | Average $\sigma_{BS} \frac{\partial \ln \sigma_{BS}}{\partial \ln S}$ |
|---|---|---|---|---|
| Total: 198 | | | -3.45 | -0.68 |
| | Golden Cross: 144 | | -3.55 | -0.62 |
| | | Negative: 112 | -5.51 | -0.98 |
| | | Positive: 32 | 3.32 | 0.66 |
| | Black Cross: 54 | | -3.21 | -0.85 |
| | | Negative: 47 | -4.08 | -1.05 |
| | | Positive: 7 | 2.6 | 0.47 |

*1/14/2011 closing values of *VIX* and *SPX* are also used to estimate the monthly values backward for 12/17/2010.

**End Notes:**

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[1] *PUT* was shown to have better returns and risk adjusted returns than *BXM* index, according to Ungar and Moran (2009). Slightly out of money puts and calls are used in *PUT* and *BXM* indexes when SPX can not exactly match a strike price available at option expiration Friday's close. As a result, volatility skew gives more premium to *PUT* than the *BXM* index.

[2] CBOE designed *BXY* index to start from a nominal value 100 on 6/1/1988 (CBOE® Micro Website 2011). *BXM* and *PUT* indexes also initially started from value 100 at 6/1/1988 which is the earliest date that S&P 500 Index has exact total return data. However, *BXM* and *PUT* indexes were later extrapolated backward to start from 6/1/1986. Given the anomaly and pricing problems in the derivatives market during the October 1987 market crash, the current study chooses to start back-testing from 6/1/1988 when daily data of these indexes are used.

[3] The BNP Paribas *RBOI* (Rules-Based Overwrite Index) is such a customary S&P 500 based Buy-Write Index with daily index value available through the Bloomberg system.

[4] Goetzmann et al (2002) proposed a general passive portfolio of a stock index and European options (including writing both a call and a put at the same time) combination to demonstrate the sensitivity of Sharpe ratio on related leverage parameters. The current study is about writing either a put or a call option dynamically in the portfolio based on projection of a market trend.

[5] *BXM* and *PUT* indexes consider the effects of bid-ask spread of option prices, available strike prices, and average prices from VWAP process for roll-over on monthly option expiration Fridays (CBOE® Micro Website 2011).

[6] The *SPTR GC-LEO* is assumed to have an frictionless trading process:

- At the day of a *SPTR* Golden Cross (when 50DMA ≥ 200DMA happens the first day in *SPTR* daily close price stream), a long position in *SPX* index is entered at the market close by liquidating all 3-month US Treasury Bills;
- Long position in *SPX* (with dividend re-invested daily) is held in days following a Golden Cross as long as *SPTR* 50DMA stays above 200 DMA;
- At market close of the day of a Black Cross (50DMA < 200DMA), the *SPX* position is sold completely and invest all proceeds in 3-month US Treasury Bills (with interest re-invested daily);
- The 3-month Treasury Bills are held as long as 50DMA stays below 200DMA for the *SPTR* Index.

[7] For *ALOOP* portfolios, the option premium collected at contract initiation are also assumed invested in the underlying equity index and Treasury bills – just as in *BXM* and *PUT* indexes.

[8] This is a more general case that allows different leverage levels for the bullish and bearish states. Yang (2010) focused an *ad hoc* case for the *ALOOP* portfolio that has one factor of leverage ($F_L$=$F_S$).

[9] The 20% margin limit on *SPX* index options is introduced here conservatively. *CBOE* specifies the minimum initial margin for uncovered written *SPX* put options at only 15% of the contracts' face value plus option premium (http://www.cboe.com/products/indexopts/spx_spec.aspx). *VIX* needs to be at about 50 for the total 20% margin limit to be hit. The *ALOOP* portfolio can reduce the *SPX* put option contracts written to reduce market exposure in high implied volatility environment and to satisfy the margin requirement. For example, for maximum $F_L$=0.8, 20% less *SPX* put contracts written in *PUT* for the *ALOOP* allows *VIX* going up to about 100 without causing a margin problem.

[10] The methodology of calculating *BXM* and *PUT* indexes already considered conservatively about option spreads, see Whaley (2002) and Ungar & Moran (2009). Hill et al (2006) pointed out as a conservative estimate that one volatility percentage point (corresponding to $f$ ~*10bp* monthly on average) can cover option spreads in monthly trading of index writing portfolios. Commission free trading on S&P 500 index based ETFs was also initiated at various brokerages in 2010 which can make costs of portfolio rebalance

and dividend reinvestment cheaper. Overall, *f=5bp* is achievable with relative large portfolios with value greater than one million USD (Yang, 2010) and *f=20bp* could represent total transaction cost plus a 2% annual fee charge on the portfolio.

[11] Here put-call parity is assumed valid. Further, any potential American calls or puts involved (e.g. ETF options are used rather than *SPX* options) are assumed close to European options when underlying equity index price or strike price is much higher than zero and underlying index has only a small dividend yield.

[12] This relates to the common perception of *VIX* as a market fear gauge. When market index S&P 500 has a large decline, *VIX* shoots higher - market participants drives up option prices by demanding more *SPX* put protection on fear of further losses.

[13] As an out-of-sample back-test, *Delta trades* criteria use an *ex ante* estimation of volatility skew (the change of *VIX* relative to the change in *SPX* index) and its impact on portfolio market exposure for the remainder of option month. In contrast, *ex post* estimation of volatility skew is used in the Appendix to measure volatility skew from historical data. The back-test is implemented as *Delta trade* only happens once in an option month on the first instance satisfying the criteria. During the full back-test period (1/19/1990-12/31/2010), there is not any instance of Golden Cross/Black Cross signal change following a *Delta trade* in the same option month. On the next option expiration Friday following a *Delta trade*, *SPTR GCBC-ALOOP* renews its positions as dictated by the Golden Cross/Black Cross signal.

[14] For example, OTC Variance Swaps for S&P500 Index and CBOE *VIX* futures contracts can be used to trade on the implied-realized *SPX* volatility spread.

[15] For example, Hill *et al* (2006) rebalanced buy-write portfolio on Thursdays before monthly option expirations. Baehr (2010) used 200-day simple moving average crosses as signals to trade the active buy-write strategy.