DE–MYSTIFYING THE "GREEKS"

The Essential Elements of Option Pricing

This article is a part of a series published by R.J. O'Brien on risk management topics for commercial agri-business clients.

In recent years, the use of options by commercial agri-business traders has increased significantly. Along with this growth has come an increase in the availability and sophistication of option pricing software. While most commercial traders have access to some form of option pricing model, often these models are not being used to their full potential.

The purpose of this article is to provide a straightforward guide to the use of option pricing models and some practical examples of how these models can be used to enhance trading decisions. In particular, this article strives to bring to life the key measures of option risk known as the "Greeks". We do this by taking an intuitive approach to the Greeks rather than focusing on the underlying mathematical formulas.

The article begins with a few words on the purpose of option pricing models and their conceptual foundations. Next, five of the most important Greeks (delta, gamma, theta, vega and rho) are discussed, along with examples of how they can be used to measure the risks and payoffs associated with various option strategies. We conclude with a discussion of the concept of an options "book", which serves to net out the various risk/return measures across a number of individual positions.

I. A BRIEF WORD ON OPTION PRICING MODELS

When dealing with most exchange-traded options, pricing models are rarely used to determine the fair market value of the option. Clearly, there is not much point in calculating the theoretical value of an exchange-traded option since the market has already established the fair market value of the option. Besides, the only way our calculated theoretical value will differ from the market price of the option is if we use a different assumption for implied volatility.

Option pricing models *are* used to estimate the fair market value when the option is so thinly traded that there is no current indication of its value (and hence its implied volatility) or for more complex, over-the-counter options for which there are no public price quotes. However, in both cases, the calculated value of the option still depends heavily on the assumption you are using for implied volatility.

For liquid, exchange-traded, options-on-futures, option pricing models are used primarily to help assess the impact of *changes* in the option pricing determinants such as the underlying futures price, days to expiration, implied volatility, etc. In other words, we use the model to help us get a better feel for what will happen to our option position given various changes in these pricing determinants. Thus, we are more concerned with *relative* changes rather than trying to determine the *absolute* value of a given option.

An option pricing model is particularly useful when our position consists of a number of different options (which is usually the case for commercial traders). In such cases, the model helps us determine what our *net* position is in terms of flat price exposure (delta and gamma), time decay (theta), implied volatility (vega) and other factors.

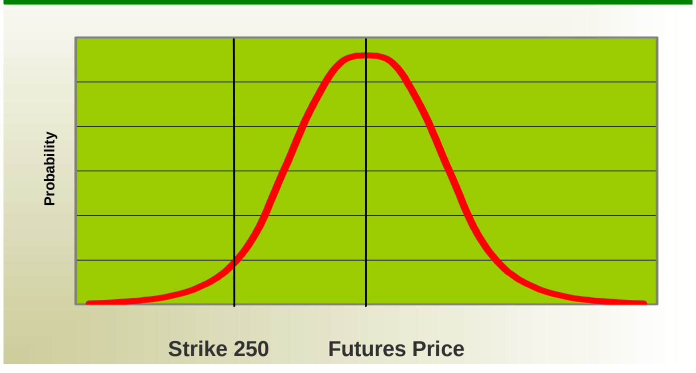
Foundations of Option Pricing Models…

Regardless of which underlying model our program is based on (Black, Cox-Ross-Rubinstein, Whaley, etc.), all option models assume that prices are random in nature. In other words, these models are *always* neutral with respect to price direction. Most models also assume that prices are log normally distributed around the mean (the familiar bell-shaped curve).

Once we know the shape and size of the probability distribution for the underlying futures price, it is a relatively simple matter for the model to calculate the theoretical value of the option. The two main determinants of the size of this probability distribution are implied volatility and the number of days to expiration. Implied volatility is actually just a form of standard deviation, which tells us the range within which prices are expected to vary over a 12 month period. The number of days to expiration simply adjusts this annual value to correspond to the actual number of days to expiration remaining in the option.

Figure 1 shows a hypothetical probability distribution for an in-the-money call. The pricing model simply calculates the theoretical value (or expected value) of the option by taking the payoff at every possible price outcome and multiplying it by the probability of that outcome[1]. In this example, all price outcomes below 250 would have a zero payoff and all price outcomes above 250 would have a positive payoff.

Figure 1. **In-the-Money Call**



When we use a model to measure the impact of changes in the futures price, days to expiration and implied volatility, we are essentially just changing the size of the probability distribution and/or moving its position around relative to the strike price.

In summary, the primary use of an option pricing model for exchange-traded options is to help quantify our risk in relation to changes in flat price, time and volatility.

II. PRICING THE TIME PREMIUM

Although the price of an option consists of both intrinsic and extrinsic (or time) value, the option pricing problem is essentially all about pricing the extrinsic value. We know that an option premium will be worth at least its intrinsic value, otherwise there will be an arbitrage opportunity. For example, a 350 put with the futures at 340 must be worth at least 10 cents. What we don't know is how much extrinsic or time value is justified at any given time.

We also know that options that are at-the-money (ATM), have the greatest time value and options that are deep out-of-the-money (OTM) or deep in-the-money (ITM) have little or no time value. It is intuitively obvious why a deep OTM option has no time value, since we can foresee no possibility of the underlying futures price moving enough to put the option in a payoff position. This is particularly true if the option doesn't have much time to expiration.

[1] Recall that the expected value of a financial instrument is equal to the sum of the payoff at each outcome multiplied by the probability of each outcome or, Expected Value = Sum of (P*$), where P is the probability of the outcome and $ is the payoff.

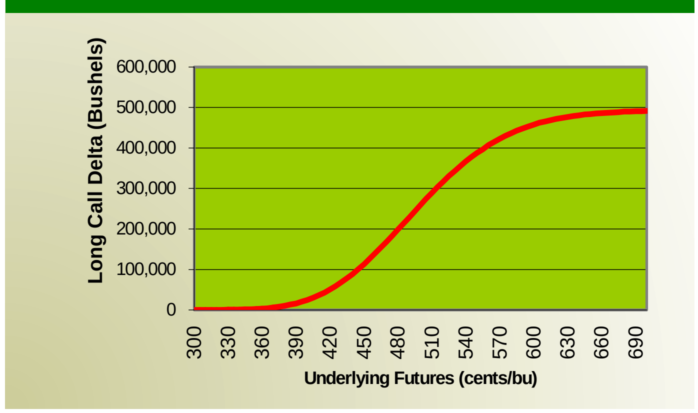
But why doesn't a deep ITM option have any time value? Isn't there still some probability that it will move deeper into the money? While there is clearly a possibility that the option will move deeper into the money and hence gain more value, there is an equal likelihood that the option will lose some of the intrinsic value that it has already gained. In other words, an option that is deep ITM behaves just like the underlying futures contract and thus warrants little or no time premium. In short, both deep OTM and ITM options have lost their option-like characteristics.

III. DELTA Δ

If you are going to master one "Greek", then it should be delta. Delta is the key measure that we use to define the flat price characteristics of an option position. Unless options are deep ITM, they do not move one-for-one with the underlying futures. **Delta is the change in the option premium given a one unit change in the underlying futures.** For example, a delta of 0.6 for a corn call means that for every one cent per bushel increase in the underlying corn futures, the call premium will increase by about 0.6 cents per bushel (and vice versa). Logically, futures positions will always have a delta of either +1.0 (long futures) or -1.0 (short futures).

As options move to a position of deep OTM, their delta approaches zero, reflecting the fact that the option premium is relatively insensitive to changes in the underlying futures. As options move deep into the money, their delta approaches 1.0, reflecting the fact that they are now behaving like the underlying futures, and hence their value fluctuates one-for-one with the underlying futures. Options that are at-the-money have a delta of 0.5, meaning there is a 50 percent probability they will move into the money sometime prior to expiration (and an equal probability they won't). Figure 2 illustrates how delta changes as a call option moves from being deep OTM to deep ITM.

Figure 2.
Relationship of 500 Soybean Call Delta to Underlying Futures
(100 Contracts/Strike Price of $5/bushel)



A more intuitive way to think of delta is in terms of the total flat price exposure they represent. If we are long 100 May soybean puts (500,000 bushels worth) with a delta of -0.60, our option position is equivalent to being short 300,000 bushels of May soybean futures. In other words, the gain or loss on these options will be similar to what we would experience if we were short 300,000 bushels of May soybean futures. If we are short 200 December corn calls (1,000,000 bushels) with a delta of 0.75, our option position is equivalent to being short 750,000 bushels of December corn futures.

If our position consists of more than one option, then it is a simple matter of calculating the delta-equivalent of each individual option and then summing them in order to calculate the net flat price exposure our position represents. Table 1 provides an example of how four soybean meal options can be combined into a single delta-equivalent or flat price position. In this example, the combined position has a net delta of long 2,490 short tons.

Table 1.

Position Delta Δ

	# of Contracts	Option	Delta	Delta Equivalent (short tons)
Long	50	July 135 Calls	0.55	2,750
Short	40	July 140 Calls	(0.39)	(1,560)
Short	100	July 125 Puts	0.13	1,300
		Net Position		**2,490**

However, since delta changes as the option moves in and out of the money, the delta-equivalent of our option position is also constantly changing. Sometimes the goal of a trader is to be delta-neutral, which means that they want the combined delta-equivalent of their futures and options positions for a given commodity to sum to zero. In other words, they don't want to be net long or short in a flat price sense. For example, a trader might buy 100 September wheat puts with a delta of 0.5 (the equivalent of being short 250,000 bushels) and then immediately buy 50 September wheat futures (250,000 bushels) to get "delta-neutral". As the delta of the option changes, they would then buy or sell futures to stay delta-neutral. The rationale for doing this is that they want to be long wheat volatility, but remain neutral with respect to flat price.

Delta is also important from an accounting perspective since it helps to quantify the amount of "coverage" our options represent. If we are long 1,000,000 bushels of November soybean calls (200 contracts) that are $2.00 per bushel out-of-the-money, does it make sense to say we have coverage of 1,000,000 bushels? Certainly not, since these options will provide very little protection unless soybean prices increase rather dramatically. It is far more accurate to state this coverage in delta-equivalent terms. For example, if these call options have a delta of 0.1, then we would show this as coverage of only 100,000 bushels. By stating the options in delta-equivalent terms we can also easily combine our option position with our futures and cash positions to determine our net coverage/exposure.

IV. GAMMA γ

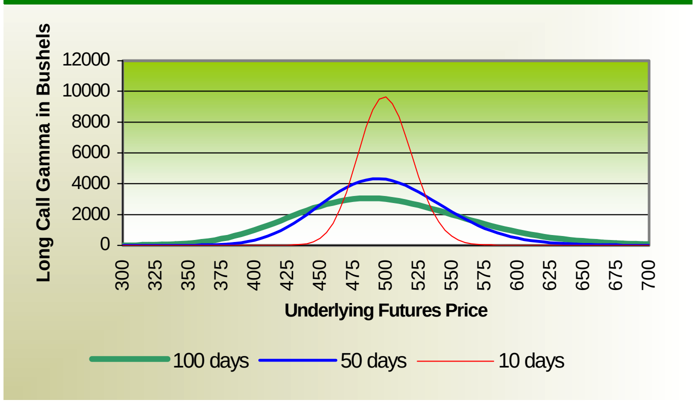
The other Greek that helps us define our flat price risk is gamma. Gamma is the rate of change of delta. **Gamma tells us how the delta-equivalent of our option position changes as the underlying futures changes.** As shown in Figure 2, the relationship between the underlying futures and the option's delta is not linear. As we move from being deep OTM to deep ITM the delta goes from zero to 1.0. Gamma is smallest for deep OTM and ITM options (for both puts and calls), reflecting the fact that the delta is essentially "locked" at zero and 1.0, respectively. Gamma is largest when the option is ATM.

As with delta, gamma is most intuitive when it is expressed in terms of its position-equivalent. For example, if we are long soybean calls with a delta-equivalent of 300,000 bushels and a gamma of 0.006, then for every one cent increase in the underlying futures, our long delta position will increase by an additional 1,800 bushels (300,000 * 0.006). If we are long options (calls or puts), gamma is always working for us in the sense that as the futures price moves in our favor we make money at an increasing rate and when the futures price moves against us, we lose money at a decreasing rate.

For example, if we are long puts, as the futures price increases, our puts move further OTM and the short delta-equivalent of our position gets smaller. Conversely, as the futures price decreases, our puts move further into the money and the short delta-equivalent of our position increases. In return for being "long gamma" we must incur the time decay that also comes along with long option positions. If we are short options (puts or calls) we are "short gamma" in the sense that our position will lose money at an increasing rate and make money at a decreasing rate.

Figure 3 illustrates the relationship between gamma and the number of days to expiration. Gamma is lowest for options that are far from expiration and highest for options that are near expiration. Not surprisingly, the acceleration of gamma as we near expiration corresponds with the acceleration of time decay. As a long option holder, although our leverage increases as we near expiration due to a higher gamma, we essentially pay for it through a greater rate of time decay.

Figure 3.
Relationship of 500 Soybean Call Gamma to Days to Expiry



So what's the practical significance of gamma to the trader? First, an understanding of the trade-off between gamma and time decay can help us fine tune our option strategy. While traders are often focused on minimizing their time decay, there are times we should be willing to incur a higher rate of time decay in return for a higher gamma.

For example, if we want to be long puts and believe that a decrease in the price of the underlying futures is imminent, we will likely favor puts that are nearer expiration since they have a higher gamma. If we are bearish, but have less conviction with respect to when the price move will occur, we may prefer to buy puts with more time to expiration and a slower rate of time decay (as well as a lower gamma). Table 2 illustrates the tradeoff between gamma and the rate of time decay (expressed in dollars per day) for an ATM corn put option given 20, 60, 100 and 140 days to expiration.

Gamma is particularly important to delta-neutral traders as it defines a large part of their risk. A trader that is short options (puts or calls) on a delta-neutral basis is short both volatility and gamma. As the underlying futures price moves higher he is getting shorter (or less long) and as the price moves lower he is getting longer (or less short). One of the major risks associated with this type of position is that the underlying futures price may move so quickly you are unable to rebalance the position in a timely fashion, thereby exposing you to considerable flat price risk. Gamma helps quantify this type of risk.

Table 2.
Gamma Versus Time to Expiration

Days to expiration	Gamma (bu)	Theta /Day
20	14,773	$667
60	8,475	$381
100	6,532	$291
140	5,479	$243

ATM Corn Puts
(100 contracts)
CZ= 230
IV=25%

V. THETA θ

Theta is the Greek that quantifies the rate at which time decay is occurring in an option. **Theta is the ratio of the change in the option premium given a one unit (usually one day) change in time.** If time decay occurred in a linear fashion, we could simply take the amount of time value in the option and divide it by the number of days to expiration to determine the daily time decay. However, as illustrated in Figure 4, time decay is not linear and accelerates as the option approaches expiration.

Theta is easily understood when expressed in a position-equivalent such as dollars per day. Table 3 shows how the rate of time decay can vary according to the amount of time to expiration and whether the option is at, in or out-of-the-money. Note that the rate of time decay (theta/day) is greatest for the ATM option that is nearest to expiration. It is also interesting to note that for the OTM and ITM options, the theta/day actually increases as we go from 10 to 50 days to expiration. The reason for this is that initially as we get further from expiration, the total amount of time value in the option increases (and therefore there is more time value to decay).

Figure 4.
Illustration of Time Decay for 500 Soybean Call

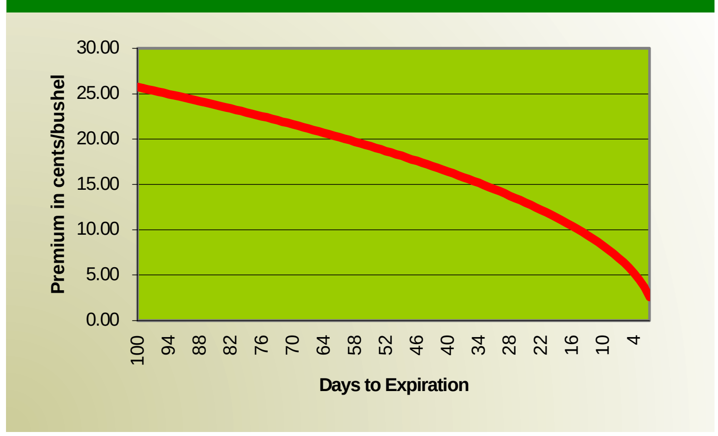


Table 3.

Time Decay (Theta)

Days to expiration	ATM (SX=525)	OTM (SX=475)	ITM (SX=575)	
	Daily Time Decay			525 Nov Soybean Calls (100 contracts) IV = 27%
10	($2,333)	($181)	($274)	
50	($1,031)	($599)	($685)	
100	($718)	($544)	($585)	

If we are long options then theta is working against us (we have a negative theta). If we are short options, then theta is working for us, since we are earning the daily time decay (we have a positive theta). If our position consists of a number of options, then a pricing model can be useful in determining whether we are net earners or payers of time premium. For example, the following position shown in Table 4, consists of both long and short options, but on a net basis has a negative theta to the tune of $224 per day.

Table 4.

Position Theta θ

	# of Contracts	Option	Theta ($/day)	
Long	50	July 135 Calls	($175)	Soybean Meal Options (SMN=136) (SMU=140)
Short	50	July 120 Puts	$53	
Long	120	Sept 140 Calls	($255)	
Short	120	Sept 125 Puts	$153	
		Net Theta	($224)	

Often hedgers like the idea of using options rather than futures to manage their price risk, but are reluctant to incur the full time decay associated with outright long option positions. An option pricing model can help to evaluate alternative strategies in relation to time decay. For example, a buyer of soybean meal might wish to consider four strategies for placing a long hedge using options (long ATM calls near expiration, long ATM calls far from expiration, long collar and ratio call spread). Table 5 shows how the rate of time decay varies according to which strategy is adopted.

Note that the strategy with the least time decay is the collar ($84/day). However, the protection provided by the collar is quite different from the other strategies and warrants careful consideration. Note also how the Dec ATM calls decay at roughly half the rate of the July ATM calls because they are further from expiration. Obviously, there is much more to this decision than time decay, as each strategy has unique flat price and volatility characteristics as well. However, being able to quantify our time decay puts us in a much stronger position to evaluate each strategy.

Table 5.

Soybean Meal Hedging Strategies

	# of Contracts	**Option**	**Position Equivalent (short tons)**	**Theta ($/day)**
A Long	100	July 135 Calls	5,526	($392)
B Long	100	Dec 145 Calls	4,999	($217)
C Long	100	July 145 Calls	2,522	($321)
Short	100	July 125 Puts	1,533	$237
Net			4,055	($84)
D Long	200	July 145 Calls	(5,526)	$392
Short	100	July 135 Calls	5,044	($641)
Net			(483)	($249)

Hedgers often regard the full option premium paid as being the "cost" of using long puts or calls in their hedging strategy. However, in many cases options are not held to expiration and thus the true "cost" of the strategy is something less than the full premium. For example, a corn feeder may want to buy calls, but only intends to hold these calls until the corn crop has successfully made it through its critical pollination stage. An option pricing model can be used to quantify the amount of time decay that will occur during the period the option is expected to be held.

For example, an ATM call option with a strike of $3.00 per bushel, 120 days to expiration and 25% volatility is worth 16 ¾ cents/bushel. However, if we only intend on holding this option for 60 days, then the amount of time decay that will occur over this period is only about 4 ¾ cents per bushel (holding everything else constant).

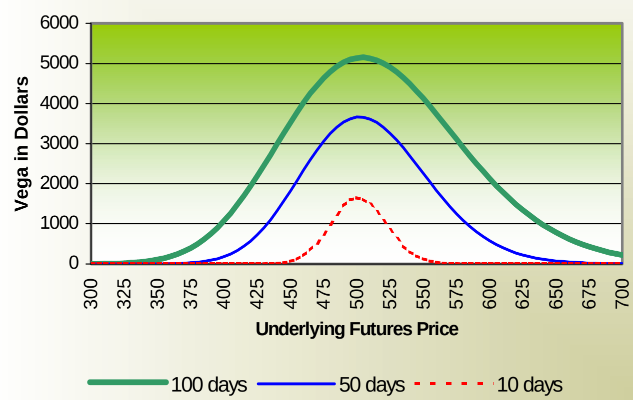
VI. VEGA ν

Vega is the Greek that quantifies our exposure to changes in implied volatility. **Mathematically, vega is the ratio of the change in the option premium given a positive one percentage point change in implied volatility.** If we are long options (puts or calls) then we are long volatility (meaning we benefit if implied volatility increases). If we are short options then we are short volatility. *Note that vega is not implied volatility, but rather the sensitivity of the option premium to changes in implied volatility.*

As with the other Greeks, vega is most intuitive when expressed in a position equivalent. For example, if our position has a vega of + $9,000, then this means for every one percentage point increase in implied volatility (i.e., going from 16% to 17%), our position will increase in value by $9,000 (and vice versa).

Unlike gamma and theta, vega is greatest for options that are far from expiration and declines as we approach expiration. This is intuitive in that the more time we have to expiration, the greater the impact of any change in implied volatility since it has more time to work. Figure 5 shows the relationship between vega, time to expiration and the underlying futures price.

Figure 5.
Relationship of 500 Call Vega to time and Underlying Futures



Vega is a great tool for helping us understand the volatility implications of positions we are considering. Often traders are surprised to find out how sensitive a position can be to changes in implied volatility. If we have a number of options in our "book", then an option pricing model can help us determine our *net* exposure to implied volatility, as illustrated below in Table 6. In this example, the net vega of our position is $7,693, which means that the value of our combined position will increase by $7,693 for every one percentage point increase in implied volatility (and vice versa).

Table 6.
Position Vega

Soybean Meal Options (SMN=136) SMU=140)

	# of Contracts	Option	Vega 1/
Long	200	July 135 Calls	$4,397
Short	200	July 120 Puts	($1,314)
Long	350	Sept 140 Calls	$11,096
Short	350	Sept 125 Puts	($6,486)
		Net Vega	$7,693

1/ $ per one percentage point change in implied volatility (100 basis points)

Take the case of a soybean oil buyer that wants to be long oil calls but wants to limit his exposure to changes in implied volatility. Table 7 below shows how the exposure to changes in implied volatility changes depending on the strategy (long-dated ATM calls, short-dated ATM calls, and short-dated OTM calls). Note that the vega is highest for the ATM calls that are furthest from expiration ($2,347) and smallest for the OTM calls that are nearer to expiration ($1,022).

Table 7.
Soybean Oil Hedging Strategies

(BON=20.37)
(BON=20.75)

	# of Contracts	**Option**		**Vega 1/**
A Long	75	Oct 2100 Calls	ATM	$2,347
B Long	75	July 2050 Calls	ATM	$1,470
C Long	75	July 2250 Calls	OTM	$1,022

1/ $ per one percentage point change in implied volatility (100 basis points)

VII. RHO ρ

Rho quantifies the relationship between interest rates and the option premium. **In other words, rho is the ratio of the change in the option premium given a one percentage-point change in interest rates (100 basis points).** For commodity options, the higher the interest rate, the lower the price of the option. This is because the buyer of the option is paying the premium to the seller up front and forfeiting the time value.

However, for practical purposes, the impact of changes in interest rates is minimal compared to the other factors discussed above and can usually be ignored (unless you are expecting a huge move in interest rates).

VIII. BUILDING AN OPTION BOOK

As we have discussed, the real power of an option pricing model becomes evident when we are able to net out our risks into a single "book". This allows us to do a meaningful comparison of alternative strategies and to make an informed decision based on our view of implied volatility, flat price direction, as well as the timing of the expected price move. While we may have accumulated a position consisting of a number of different strikes and contract months, we are usually most interested in our net position.

Table 8 shows a book of wheat options that a long hedger might have accumulated over a period of time. On a net basis, this position is long 1,207,943 bushels (delta-equivalent), net short volatility (vega of -$2,292), and is incurring time decay at the rate of $ 970 per day.

In addition to taking a snapshot of each position, an option pricing model can be used to evaluate a position under a wide range of possible outcomes. This is something we will discuss in future articles.

- Ron Gibson

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Table 8.

Building an Option Book

Long or Short	Option	# of Contracts	Delta	Position Equivalents: Delta (bushels)	Gamma (bushels)	Theta ($/day)	Vega ($/point)
long	wn260c	30	0.62	92,904	1,871	(142)	608
long	wn270c	23	0.49	56,512	1,436	(121)	490
long	wn290c	140	0.28	196,520	6,998	(669)	2,529
long	wn290c	50	0.55	136,307	1,683	(123)	2,144
long	wz310c	50	0.42	105,699	1,566	(135)	2,141
short	wz280p	200	0.36	24,624	(6,493)	477	(8,202)
short	wz270p	100	0.29	145,295	(3,000)	221	(3,778)
long	kwn310c	20	0.35	35,313	1,036	(112)	437
long	kwn330c	80	0.2	81,768	2,966	(366)	1,338
	NET POSITION			**1,207,942**	**8,063**	**(970)**	**(2,293)**

Note: All option calculations in this article have been made using RJO's PositonBook© software. This product is provided as a service to RJO's commercial agri-business clients.