IMPLIED VOLATILITY SURFACES: UNCOVERING REGULARITIES FOR OPTIONS ON FINANCIAL FUTURES†

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ABSTRACT

It is well known that the implied volatilities of options on the same underlying asset differ across strike prices and terms to expiration. However, the reason for this remains unclear. Before the development of theory to explain this phenomenon, it may be helpful to better understand the empirical record of implied volatility surfaces. If regularities are discovered which are stable overtime, this may aid the development of theories to explain implied volatility surfaces and provide a means to test alternative models. This paper identifies these regularities and subsequent research will examine the implications of these results.

While a number of papers have examined individual option markets and identified smile patterns, it is not clear whether the conclusions found are based upon idiosyncrasies of a particular market or more generally apply to options in other markets. This research fills this gap in the literature by examining sixteen options markets on financial futures (comprising four asset classes) and compares the smile patterns across markets. Furthermore, this analysis considers a longer period of analysis than previously examined in the literature. This allows assessment of the stability of the implied volatility patterns for a variety of subperiods and testing of models outside of sample.

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1. INTRODUCTION

If the Black-Scholes-Merton model [Black and Scholes (1973) and Merton (1973)] accurately describes conditions in traded options markets, then the volatilities implied from the market price for options on the same underlying asset would be identical regardless of the strike price of the option or its maturity. However, it is now generally accepted that the implied volatilities differ across strike prices for the same maturity and across diverse expiration periods.[1]

A recent trend in the empirical investigation of implied volatilities has been to concentrate on understanding the behaviour of implied volatilities across strike prices and time to expiration [see Jackwerth and Rubinstein (1996)]. This line of research assumes implicitly that these divergences provide information about the dynamics of the options markets. Another approach [Dupire (1992, 1994), Derman & Kani (1994) and Rubinstein (1994)] suggests that the divergences of implied volatilities across strike prices may provide information about the expected dispersion process for underlying asset prices. These papers assume that asset return volatility is a [locally] deterministic function of the asset price and time and that this information can be used to enhance the traditional option pricing approach of Black & Scholes (1973). All these papers examine implied volatility patterns at a single point in time and assume that option prices provide an indication of the deterministic volatility function. Recently, Dumas, Fleming and Whaley (1996, 1998) tested for the existence of a deterministic implied volatility function and rejected the hypothesis that the inclusion of such a model in option pricing was an improvement in terms of predictive or hedging performance compared with Black and Scholes (1973). Their research examined whether at a single point in time, implied volatility surfaces provide predictions of implied volatilities at some future date (one week hence).

This research looks at a different problem. The Dumas, Fleming and Whaley (1996,1998) approach assumes the deterministic volatility function provides both a prediction of the future levels of implied volatility and the relative shapes of implied volatilities across strike prices and time. If the future levels of implied volatilities cannot be predicted, this does not mean that the relative shapes of implied volatilities cannot be predicted. Regularities in smile surfaces may be undiscernable when expressed in levels, which may follow an unpredictable stochastic process[2]. Once the level of the implied volatility is controlled for, we find considerable regularities in the relative shapes of implied volatility surfaces.

Secondly, Dumas, Fleming and Whaley (1996,1998) examine whether the levels of implied volatilities for options with a certain time to expiration (i.e. 90 days) will predict the levels of volatilities of options with a different time to expiration (i.e. 83 days). If the relative shapes of implied volatility surfaces depend on the time to expiration, this might be the wrong comparison. This research will compare the relative shapes of implied volatility surfaces for all options with the same time to expiration. Given the substantial data available for this research, this will allow enough observations to test whether option surfaces with the same time to expiration display regularities. As with the previous problem, the levels of implied volatilities are controlled for by the standardisation of the implied volatility surfaces.

If implied volatility surfaces (with the same term to expiration) retain the same shape over time by structuring the problem in this manner, this could lead to a better understanding of why smiles exist. As with Rubinstein (1994), the primary motivation of this research is to assign economic significance to the functional form of the smile patterns. In addition, it is necessary to test the stability of this functional form over time for individual markets. A second motivation was to broaden the scope of this research by comparing a wide range of options on financial assets. To date, most research has concentrated on options on one asset [see Corrado and Su (1996) that looked at the S&P 500] or options on a relatively small number of assets in the same asset class [see Corrado & Su (1997) that examined stock options on US stocks]. Therefore, this research fills a gap in the literature with a cross-sectional analysis of implied volatility surfaces for sixteen financial markets drawn from four asset classes: stock indices, bonds, foreign exchange and forward deposits. Furthermore, this analysis will be done for a longer period of analysis than has previously been examined in the literature, allowing assessment of the predictive ability of the implied volatility functional form.

The paper is organised as follows. The second section will discuss the data sources. The third and fourth sections discuss the standardisation procedure and provide graphical results. The fifth section presents a simple model based on a simple polynomial functional form and is motivated by the empirical results in section four. This is followed by a test of this functional form using standard OLS regression that will allow the coefficients of the statistical model to quantify the strike price effects and facilitate comparisons between markets. At that point, the functional forms of the implied volatility surfaces will be tested outside of sample. Finally, conclusions and suggestions for further research appear.

2. DATA SOURCES

The options examined in this research are options on futures[3]. The following markets were examined for options on stock index futures: the S&P 500, FTSE 100, DAX and Nikkei 225. For option on bond futures: US Treasury Bonds, UK Gilts, German Bundesanleihen (Bunds), and Italian Government Bonds (BTPs) were selected. For options on currency futures, US Dollar/Deutsche Mark, US Dollar/British Pound, US Dollar/ Japanese Yen and US Dollar/Swiss Franc futures were examined. Finally, for options on 3 month Deposit Futures, the Euro Dollar, Euro Sterling, Euro D-mark and Euro Swiss Franc markets were selected.[4] For all of the option markets, the analysis period extends back in time to include all the publicly available data. In most cases more than 10 years were available. This allows us enough observations to conduct a meaningful analysis.[5] The individual options markets examined and the time periods of analysis for each appear in Table A.

Underlying Asset	Time Period of Analysis
S&P 500 Futures	25/03/1986 - 24/12/1996
FTSE Futures	02/01/1985 - 20/12/1996
Nikkei Dow Futures	25/09/1990 - 16/12/1996
DAX Futures	02/01/1992 - 20/12/1996
Bund Futures	20/04/1989 - 21/11/1996
BTP Futures	11/10/1991 - 21/11/1996
Gilt Futures	13/03/1986 - 22/11/1996
US T-Bond Futures	02/01/1985 - 15/11/1996
Deutsche Mark /US Dollar	03/01/1985 - 09/12/1996
British Pound / US Dollar	25/02/1985 - 09/12/1996
Japanese Yen / US Dollar	05/03/1986 - 09/12/1996
Swiss Franc / US Dollar	25/02/1985 - 09/12/1996
Euro Dollar	27/06/1985 - 16/12/1996
Euro Sterling	05/11/1987 - 18/12/1996
Euro D-mark	11/03/1990 - 16/12/1996
Euro Swiss Franc	15/10/1992 - 16/12/1996

Table A Time Period of Analysis of the Sixteen Underlying Assets Analysed

The data for the options and futures contracts was obtained directly from the exchange where they trade.[6] For all markets, the data obtained included the closing prices of the options, the strike prices and the price of the underlying futures recorded at the same time as the options closing price. Only futures and options contracts that were the nearest contracts to expiration and traded on the quarterly expiration schedule of March, June, September and December maturities were considered. This was done to assure that the derivative contracts were actively traded, thus reducing spurious effects due to illiquidity. For the futures and options on 3-month deposits, the prices were re-expressed as a forward interest rate by subtracting both the futures price and the striking prices of the options from 100. Given this conversion, the calls (puts) on the Deposit futures were reclassified as puts (calls) on the annualised forward interest rates.

As is standard, all options prices traded at the minimum level at the relevant market or allowed a butterfly arbitrage were excluded [see Jackwerth and Rubinstein (1996)]. Furthermore, to reduce the potential problem of nonsynchronous prices for the options and underlying futures, only those implied volatilities from the available out-of-the-money (OTM) option contracts (not admitting arbitrage) were examined. Bates (1991) and Gemmill (1991) have shown that much greater deviations occur in the implied volatilities for in-the-money (ITM) options relative to the OTM options. They suggest that this is due to futures and ITM option prices not being recorded simultaneously. Thus, if the strike price was equal to or below the underlying futures price, put options were examined; otherwise call options were examined.[7] The implied volatilities for all American style options on Futures were estimated using the Barone-Adesi and Whaley (1987) model and all European style options used the Black (1976) model. The interest rate inputs were obtained from the Federal Reserve Bank in New York (US Dollar Treasury Bill Rate) or from The Bank of England (LIBOR for all other currencies).

3. STANDARDISED IMPLIED VOLATILITY SURFACES: METHODOLOGY

While a number of approaches have been proposed to standardise implied volatilities, the simplest approach is to create an index: the implied volatilities at each strike price are expressed as the ratio to the implied volatility of the option closest to the at-the-money (ATM) level. Fung and Hsieh (1991) and Natenberg (1994) used this approach. This transformation is required because the levels of volatility are not constant over time and vary across the markets. The logic behind this approach is that the relative relationships between the volatilities and not the absolute levels are of interest. This standardised volatility will be referred to as a Volatility Smile Index (VSI).

The strike prices must also be standardised to allow comparisons to be drawn. A simple approach suggested by Fung and Hsieh (1991) and Jackwerth and Rubinstein (1996) was to take the ratio of the strike price to the underlying price (with the former inverting this ratio). While this has practical advantages for market participants (namely it is simple to reverse the equation to obtain actual strike prices), the approach is somewhat misleading as the ratios are time-independent, while options prices are not. An approach more consistent with the time-dependency of option prices is that suggested by Natenberg (1994). This was slightly modified for this research and can be expressed as:

$$\frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau / 365}} \qquad \textbf{(1)}$$

Where X is the strike price of the option, F is the underlying futures price and the square root of time factor reflects the percentage in a year of the remaining time until the expiration of the option. The sigma (σ) is the level of the ATM volatility. The inclusion of the ATM volatility allows the strike prices to be expressed in standard deviation terms.[8] For this analysis, we assumed that the relevant time is calendar days and expressed time as the percentage of calendar days remaining in the options life to the total trading time in a year (which was assumed to be 365 days)[9].

4. STANDARDISED IMPLIED VOLATILITY SURFACES: RESULTS

Using all available options prices for each of the sixteen markets, we converted the levels of the implied volatilities into index form with the ATM implied volatility as the numeraire. The ATM volatility was determined using a simple linear interpolation for the two implied volatilities of the strike prices that bracketed the underlying asset price.[10] The second standardisation was to re-express the strike prices using formula 1.

As the analysis was restricted to the quarterly expiration schedule of March, June, September and December maturities, implied volatility surfaces with a maximum term to expiration of approximately 90 days were obtained. Data was further pruned by restricting the analysis to eighteen time points from (the date nearest to) 90 calendar days to expiration to (the date nearest) 5 calendar days to expiration in 5-day increments.[11] Finally, the analysis of the implied volatilities was limited to those strike prices in the range ±4.5 standard deviations away from the underlying futures price.

To assess general patterns, all the implied volatilities (for the entire period of analysis) were aggregated into each of the eighteen time periods to expiration. With these (time to expiration) homogenous samples, the following regression proposed by Shimko (1991,1993) was run:

$$VSI = \alpha + \beta_1 \cdot \frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau/365}} + \beta_2 \cdot \left[\frac{\ln(X_\tau / F_\tau)}{\sigma\sqrt{\tau/365}}\right]^2 + \varepsilon \qquad \textbf{(2)}$$

With the coefficients from this quadratic regression a fitted line was produced for each expiration period and for each market. These results are graphed in Figures 1 for the four stock index options, 2 for the four bond options, 3 for the four foreign exchange options and 4 for the four deposit options.[12] These figures seem to suggest that consistencies in the general dynamics of the implied volatility surfaces occur within the same asset class:

- Most stock index futures options have a relatively linear skewed pattern with 90 days to expiration and become more curved the closer to expiration.
- A similar pattern is observed for options on Bond Futures.
- Even greater consistency exists among the four foreign exchange options and four deposit options markets.

These graphs are consistent with Shimko's (1991,1993) findings that a simple quadratic function will yield a smooth well-behaved curve for a cross section of implied volatilities. Furthermore, the degree of explanatory power for quadratic regression for each of the eighteen periods is comparable to those reported by Shimko (1991,1993)[13]. If the sole objective was to fit a curved line, this has been achieved. However, for our purposes, this approach is unsuitable. The quadratic regression approach was intended to fit implied volatilities not only with the same time to expiration but also contemporaneously. In our analysis, the sample of implied volatilities represents a grouping of all options with the same time to expiration but estimated on different dates. Therefore, the quadratic approach provides little (prima facie) information about the stability of the quadratic function over time. We only gain an understanding of the average relationship over the entire period of analysis. In addition, this approach only fits a curved line, whereas the goal here was to fit a curved surface. Therefore, to incorporate the time dependency of the smile surfaces, a richer fitting approach was required.

5. MODELLING STANDARDISED IMPLIED VOLATILITY SURFACES

A logical starting point for an appropriate functional form to fit an implied volatility surface is the approach suggested by Dumas, Fleming and Whaley (1996, 1998), who tested a number of arbitrary models based upon a polynomial expansion across strike price (x) and time (t). In this research, we extended the polynomial expansion to degree three[14] and included additional factors, which may also influence the behaviours of volatility surfaces. We extended the findings of Rubinstein (1994) in order to assess if implied volatility patterns changed after the 1987 stock market crash for markets other than the S&P 500. Furthermore, we examined how other individual market specific shocks might affect the shapes of implied volatility surfaces for these markets.

We considered a Taylor's series expansion to degree three. Expanding the function $\sigma = f(x,t)$ with Taylor's expansion series

$$\sigma = \sum_{i,j=0}^{\infty} \frac{1}{i!\,j!} \frac{\partial^{(i+j)}\sigma}{\partial x^i \partial t^j} x^i t^j \tag{2.1}$$

and stopping the expansion at the third degree ($i + j \leq 3$), we obtained

$$\begin{aligned}\sigma \approx {} & \frac{\partial \sigma}{\partial x} x + \frac{\partial \sigma}{\partial t} t + \frac{1}{2!} \frac{\partial^2 \sigma}{\partial x^2} x^2 + \frac{1}{2!} \frac{\partial^2 \sigma}{\partial t^2} t^2 + \frac{1}{1!1!} \frac{\partial^2 \sigma}{\partial x \partial t} xt + \\ & + \frac{1}{3!} \frac{\partial^3 \sigma}{\partial x^3} x^3 + \frac{1}{3!} \frac{\partial^3 \sigma}{\partial t^3} t^3 + \frac{1}{2!1!} \frac{\partial^3 \sigma}{\partial x^2 \partial t} x^2 t + \frac{1}{1!2!} \frac{\partial^3 \sigma}{\partial x \partial t^2} x t^2\end{aligned} \tag{2.2}$$

Given we had nine derivatives in the expansion, we constructed nine variables to capture these effects[15]. In equation 2.2, σ is the standardised volatility (VSI), x is the strike price expressed in terms of equation 1 and t is the term to maturity (in percentage of a year). To capture the effect of the 1987 crash and market specific shocks, a dummy variable was constructed, assuming a value of 0 prior to the event and 1 thereafter. To determine when such a shock had occurred, we examined the exponentially weighted unconditional volatility time series for each market. The first two most extreme spikes in the unconditional volatility that occurred during the periods (see Table A) were chosen as shock events. The dates of these two shocks appear in Table B.

Underlying Asset	First Shock	Second Shock
S&P 500 Futures	19/10/1987	13/10/1989
FTSE Futures	19/10/1987	16/10/1989
Nikkei Dow Futures	21/08/1992	07/07/1995
DAX Futures	05/10/1992	02/03/1994
Bund Futures	21/02/1990	13/06/1994
BTP Futures	05/10/1992	16/06/1994
Gilt Futures	30/09/1986	02/06/1994
US T-Bond Futures	09/06/1986	28/04/1994
Deutsche Mark /US Dollar	23/09/1985	21/08/1991
British Pound / US Dollar	23/09/1985	16/09/1992
Japanese Yen / US Dollar	23/09/1985	05/01/1988
Swiss Franc / US Dollar	23/09/1985	05/01/1988
Euro Dollar	18/12/1990	10/10/1994
Euro Sterling	16/06/1988	17/09/1992
Euro D-mark	10/09/1992	29/12/1992
Euro Swiss Franc	03/06/1997	15/09/1998

Table B, Dates on Which Two Major Shocks in the Unconditional Variance Occurred for the Sixteen Markets Under Examination.

On each of these dates, a relevant economic event was identified as the cause for the shock to the unconditional return series. To assess the impacts on the strike price effect, these dummy variables were multiplied by the first and second order strike price variables from equation 2.2. For the S&P 500 and FTSE 100 futures, these two events were the 1987 stock market crash and the 1989 mini-stock market crash. For the DAX and Nikkei (only analysed after the 1989 mini-crash), the shocks to the unconditional returns were country or market specific. For the DAX, the October 1992 shock was due to the aftermath of the EMS crisis, when a number of major German trading partners (Britain and Italy) were ejected from the exchange rate mechanism. The March 1994 shock was associated with a Bundesbank change in interest rate policy. The two shocks for the Nikkei were both associated with the index falling below the psychologically sensitive 15,000 level. Both events were associated with the release of unfavourable macro-economic data regarding the Japanese economy.

For the bond markets, the first shocks tended to be country specific. The first shock to the Bund market was related to issues regarding the re-unification of West and East Germany. The first shock for the BTP market occurred in June 1992 when Denmark rejected the Maastricht treaty in a referendum and ultimately led to the ERM currency crisis occurring in the Fall of 1992. For the Gilt market, the first major shock occurred in September 1986 and was associated with a change in monetary policy by the Bank of England. For the US T-Bond market, the first major shock occurred after a June 1986 G7 meeting (and was due to perceived conflicts between United States and Japanese economic policies). For all the bond markets, the second shock occurs in the Spring of 1994 and was associated with the pre-emptive rise in the Discount Rate by the US Federal Reserve to stem perceived inflationary pressures.

For all four currency markets, the first shock was associated with the concerted intervention in the currency markets by the Group of Seven (G7) to put pressure on the US Dollar, which was perceived as over-valued. The second shock for both the Swiss Franc and Japanese Yen occurred in January 1988 and was seen as an aftermath of the 1987 Stock Market Crash when both Swiss and Japanese investors began reducing holdings of US investments. The second shock for the British Pound came on September 16th, 1992 when the British Pound was ejected from the European Monetary System by speculative pressures. For the Deutsche Mark, the second shock occurred in August 1991 and was due to market uncertainty regarding the success of the re-unification of West and East Germany.

Many of the shocks for the deposit futures are similar to the experiences for the currencies. For example, the second shock for Euro Sterling and both shocks to Euro D-mark are associated with the expulsion of Sterling from the EMS in September of 1992. The first shock for the Euro Sterling market was associated with a change in Bank of England interest rate policy in June 1988. For the Euro Dollar, the two shocks (December 1990 and October 1994) were also associated with changes in US interest rate policy. The shocks to the Euro Swiss occurred in June 1997 and September 1998 and were associated with a massive inflow of funds from EU investors who anticipated weakness in the soon to be launched Euro.

Finally, there is concern that important information has been removed from the analysis by the process of standardising the strike prices and implied volatilities. To examine the possible contributions of these two elements, the level of the ATM implied volatility and the (natural logarithm of) futures prices were included in the model[16]. To understand the dynamics of strike price effects better, combination variables were determined [the products of the first and second order strike price effects in equation 2.2 with the level of the ATM implied volatility and the level of the log futures price].

The final form of the model can be expressed as:

$$\begin{aligned}VSI &= \alpha + STRIKE \cdot (\beta_1 + \beta_2 TIME + \beta_3 TIME^2 + \beta_4 CRASH + \beta_5 SHOCK1 + \beta_6 SHOCK2 + \beta_7 ATMVOL + \beta_8 FUTURE) \\ &+ STRIKE^2 \cdot (\beta_9 + \beta_{10} TIME + \beta_{11} TIME^2 + \beta_{12} CRASH + \beta_{13} SHOCK1 + \beta_{14} SHOCK2 + \beta_{15} ATMVOL + \beta_{16} FUTURE) \\ &+ \beta_{17} STRIKE^3 + \beta_{18} CRASH + \beta_{19} SHOCK1 + \beta_{20} SHOCK2 + \beta_{21} ATMVOL + \beta_{22} FUTURE + \beta_{23} TIME + \beta_{24} TIME^2 \\ &+ \beta_{25} TIME^3 + \beta_{26} VSi(-1) + \beta_{27} MA(1) + \varepsilon\end{aligned} \tag{3}$$

Given the model is a mixture of regular variables and dummy variables, a logical estimation procedure would be an analysis of covariance (ANCOVA) with the standardised implied volatilities (VSI) as the dependent variable. Using a standard OLS approach, problems with heteroskedasticity and serial correlation were found. While alternative regression approaches were examined to correct for each of these problems, it was deemed more convenient to present these results using the Newey-West (1987) estimator for the weighting covariance matrix. While this approach was sufficient to address problems of heteroskedasticity, problems with serial correlation remained (indicated by Durbin-Watson statistics). To address this problem, two additional variables were included in the model. The first was the lagged level of the VSI [VSI(-1)] and the second was a simply moving average of the residual terms [MA(1)]. The addition of these two terms is required to remove serial correlations that arose if the static equation [(3) without these terms] was used. Hendry & Mizon (1978) and Mizon (1995) use a similar approach. They argue that the existence of serial correlation indicates model misspecification and thus, the model must be respecified instead of adopting the faulty alternative of correcting for serial correlation. They demonstrate that this simple respecification of equation (3) from a static to a dynamic equation will yield more consistent estimates.

Given that the estimation of equation (3) is at the heart of this research, one must be sensitive to misspecification both in the structure of the model and in the estimation procedure. Such potential causes for misspecification include: 1) Omission of Critical Variables, 2) Existence of Heteroskedastistic conditional distributions, 3) Serial Correlations in the residuals and 4) Multi-collinearity in the independent variables.

One of the most challenging problems with any experimental design is the selection of the variables and the possible of critical variables. One potential variable for inclusion is the level of short-term interest rates. This was initially included in the analysis and provided only a marginal contribution for the bond and currency markets. This effect disappeared when alternative regression approaches were done. Other research by Peña, Rubio and Serna (1999) has found that the bid-ask spread of the options (as a proxy for liquidity costs) and volume related variables (in both the options and the underlying asset markets) are statistically significant when explaining the volatility smiles of IBEX options. While this research does not consider these impacts, it may prove fruitful for future research to include these variables in Equation (3). Nevertheless, the estimation of Equation (3) in the current form explains the vast majority of the variance in the implied volatility surfaces and it is possible that while these effects are significant, they are of secondary importance.

Apart from these variables, it is not obvious which other variables could be included. It is apparent that shocks do play a role in the dynamics of implied volatility processes. However, it is not clear that the shock events selected are the relevant events. Perhaps period specific events were missed. To examine if this is the case the regression was rerun including all the individual contracts from 1985 to 1996 as dummy variables. Very few of the dummy variables were significant and the inclusion of these variables does not substantively alter the estimations of the coefficients of the independent variables from Equation (3).

To assess the problem of heteroskedasticity, weighted least squares regressions were run for all sixteen markets. Initially, we followed the lines of Neter, Wasserman &

Kutner (1985) and Kvålseth (1985) to correct solely for this problem. Subsequently, the more general approach of Newey-West (1987) was used (which should address both heteroskedasticity and serial correlation). Both approaches corrected for the problem and yielded similar signs and magnitudes of the estimated coefficients of the model.

The initial approach chosen to resolve the problem of serial correlations in the residuals is the Generalised Differences approach to Generalised Least Squares (GLS). In addition, the Newey-West (1987) estimator for the weighting covariance matrix was also used. As was previously indicated, the Newey-West (1987) estimator failed to correct for the serial correlation and the model was respecified along the lines suggested by Hendry & Mizon (1978) and Mizon (1995). While a number of the variable coefficients display statistically different values using the alternative estimation procedures, none of the signs of the impacts changed. Given the primary objective of this research is to assign economic significance to the estimation of equation (3), the fact that the statistical significance of the coefficients (and the sign of the impact) are retained once alternative regression approaches are used, leads us to conclude the model is robust to the method of estimation.

Finally, by design many of the variables in the regression are highly correlated and this could possibly introduce a problem of multicollinearity. For example, it could be argued that this regression model is incorrect because the ATM volatility appears on both sides of the equation. This is because the dependent variable, the standardised volatility, is indexed to the ATM volatility and many of the independent variables also include the ATM volatility. However, this is not a serious problem as the inclusion of the ATM volatility in the dependent variable (and many of the independent variables) simply allows variables to be expressed in a standardised form[17]. For the remaining variables, this potential problem was partially addressed by the choice of a step-wise selection of variables in alternative regression approaches. Judge, et al (1980) suggest the use of a Principal component regression to address this problem. We chose not to employ this approach, as this would restrict the inclusion of the dummy variables in Equation (3) and many of these (the Crash for example) provide considerable explanatory power to the model and have important economic interpretations. Given the significance of many of the included variables and the fact that these allow useful (and consistent) economic interpretations to be drawn, the danger of risking multicollinearity is more than outweighed by the danger of losing information by omitting important variables.

Regardless of the alternative approaches for the estimation of Equation (3), it is clear that this is a fairly complex model with a large number of variables. Normally, when evaluating an equation with as many independent variables, there are too many parameters to be determined satisfactorily. However, the number of observations is extraordinarily high. In addition, the results have high degrees of explanatory power (adjusted R squared). Another problem is that over-fitting may be endemic when the number of independent variables is large relative to the number of observations and subsequently, the models may fail to predict outside of sample. To examine this issue, the models were re-estimated for the first half of the available data and this result was used to forecast the relative implied volatilities in the second half of the available data set. The models retain a high level of explanatory power outside of sample and have appropriate coefficients. This will be discussed in a later section.

6. MODEL ESTIMATION AND TESTING

For the sake of convenience, only the results using the Newey-West (1987) estimator for the weighted covariance matrix with the Hendry & Mizon (1978) and Mizon (1995) model respecification are presented. Using this approach, equation (3) was estimated for all sixteen markets, initially using all the available data. The results of these statistical procedures can be seen in Tables 1, 2, 3 and 4 for the four asset classes, stock index futures, bond futures, foreign exchange futures and 3 month deposit futures.

In these tables, the coefficients of the regression are presented along with the standard error of the estimates and the t-statistic.[18] For all variables that have a significant t-statistic (at a 95% level), the results are presented in **bold** type. All results that are not bolded were not significantly different from zero for the independent variables or from 100 for the intercept. When a particular variable was not selected in the forward stepwise regression, this is represented by "-.--". We have also included the number of observations included in the analysis, the adjusted R-squared statistic and the Durbin-Watson statistic.

6.1 STOCK INDEX OPTIONS

In Table 1, we find that most of the strike price related independent variables are statistically significant for the four stock index options[19]. Furthermore, the explanatory power of each of the models is extremely high. The adjusted R-squared statistic is between 0.9051 (for the FTSE) to 0.9682 (for the S&P). These results suggest that the models are explaining almost all the variance in the relative implied volatility surfaces. The use of the modified Newey-West (1987) estimator indicates that problems with serial correlations in residuals are not relevant.

Model interpretation begins with the first order strike price effect (skewness). For all four stock index options markets, the coefficient (of β_1) is *insignificant.* This suggests that controlling for all other variables, there is no skew effect. At first glance, this is counter-intuitive, as it is inconsistent with the smile surfaces for the stock index options represented in Figure 1. However, caution must be exercised in the interpretation of the model, as the overall strike price effects are an aggregate of a number of variables. To gauge the overall first order strike price effect; one must compare all STRIKE related variables including dummy variables associated with shocks.

For the S&P 500 and the FTSE 100 markets, the negative skew is a result of both the 1987 stock market crash and the first shock (the 1989 "mini-Crash" for the S&P 500). This confirms the finding of Rubinstein (1994) that the skew in the implied volatility smile for the S&P was only observed after the 1987 crash. However, it appears that not only the 1987 crash but also the 1989 correction contribute to the negative skewness of smiles. For the DAX and Nikkei, impacts of shocks on first order strike price effects vary. For the DAX, neither the first or second shock significantly change the first order strike price effect. For the Nikkei, the first shock caused the skewness effect to become more positive (or less negative). Nevertheless, for all four markets the smile surfaces in Figure 1 indicate negatively skewed implied volatility surfaces.

This apparent anomaly may be explained by fact that other variables cause the overall negative skewness we observe. For DAX options, the level of the underlying DAX futures has a significantly negative impact on the first order strike price effect [coefficient for this effect (β_8) of -5.2454]. For the DAX, it appears that when the futures price rises, market participant increase their assessment of the probability of future market weakness. This result is consistent with findings of Peña, Rubio and Serna (1999) for Spanish IBEX-35 index options. They also found that the higher the levels of the futures price, the more negative the skew. However, this effect is not observed for the other stock index options markets.

For both the Nikkei and S&P 500 options markets, negative skewness is associated with the level of the at-the-money volatility [coefficient for this effect (β_7) of –

11.9768 and -12.6910, respectively]. This is contrary to results found by Peña, Rubio and Serna (1999) for IBEX-35 index options. They report that the degree of a negative skew is inversely related to the level of the ATM volatility. A similar result is found FTSE options (coefficient for β_7)]. This suggests that the lower the level of the ATM implied volatility, the higher the degree of the negative skew. Why these effects differ from those reported by Peña, Rubio and Serna (1999) remain unknown and may be an important question for future research. This might suggest that some common and systematic effect occurs for European stock markets may exist (DAX options also have a positive effect, but not significant).

The effects of the interaction variables that combine STRIKE and TIME (coefficients β_2 and β_3) are more consistent across the four stock index options markets. The negative coefficient for β_2 and the positive coefficient for β_3 imply that as the expiration of the option is approached the degree of negative skewness is reduced.[20] To compare these effects properly, it is important to realise that the overall effect of time is a combination of both variables. However, the signs and magnitudes of the effects are similar, leading to a conclusion that the time dependency of the first order strike price effect is general for all four stock index options.

The second order strike price effect (curvature) appears to be much more consistent between the four stock index option markets (coefficients β_9 to β_{16}). When the variables have significant coefficients (apart from shock dummy variables) the sign of the effect and the levels of significance are similar. For all four markets, the Beta coefficient for the pure curvature effect ($STRIKE^2$) is positive. The first order impact of $STRIKE^2$ with TIME is negative for all the models and for the second order time dependent impact is positive (when significant). This suggests that the curvature of the surfaces becomes more extreme as expiration is approached.

The time effects for the curvature are opposite to those found for the skewness. The degree of skewness becomes more negative as the more the time to expiration, while the degree of curvature becomes less extreme. This is somehow counter-intuitive as one would expect the degree of negative skewness to increase with the degree of curvature [see Duque and Teixeira, (1999)]. This result suggests that a rational explanation for the existence of smiles based upon either non-normal i.i.d. price processes or subordinated stochastic volatility models would be precluded. The implications of these findings are being considered in ongoing research, which examines alternative hypotheses to model smile behaviour. This apparent counter-intuitive result is an important clue as to the choice of appropriate models, which are consistent with observed smile patterns.

The impacts of the 1987 crash and market specific shocks have different effects for different markets. For the S&P 500, the 1987 crash led to a small (but insignificant) increase in the level of curvature. Contrary, for the FTSE, the curvature was significantly reduced after the crash. For both the S&P 500 and the FTSE options, the 1989 mini-crash actually reduced the level of curvature. Neither of the shocks changed the curvature for the DAX or Nikkei options. Thus, we conclude that the curvature in implied volatility surfaces predates the 1987 crash and does not appear to change in a systematic manner as market shocks occur.

Two variables that may prove fruitful in fostering our understanding of smiles are the relationships between the levels of the expected variance and between the underlying asset. For all four stock index options, there is a significantly negative coefficient for the relationship between the level of the ATM volatility and the degree of curvature (β_{15}). This suggests that the higher the level of the expected variance, the flatter the curve of the implied volatility pattern. This result was also observed by Peña, Rubio and Serna (1999) for IBEX-35 index options. Furthermore, the level of the futures has an inverse relationship to the curvature of the implied volatility smile for DAX and Nikkei options [this fact is also pointed out by Peña, Rubio and Serna (1999) for IBEX options].

The final consistent effect for all four stock index options are the third order strike price effects. The coefficient for STRIKE^3 variable (β_{17}) is positive for all four markets and roughly of the same order of magnitude. One interpretation of this result is that the degree of curvature of the implied volatility pattern is higher above the level of the underlying futures and lower below. This is consistent with the inverted "J" shape of the implied volatility surfaces observed in Figure 1.

The other non-strike price related variables are for the most part insignificant apart from the time-related variables. The fact that the intercepts of the regression are statistically different from 100 might indicate the existence of errors in the determination of some of the variables. However, alternative regression approaches reduce or eliminate many of these effects without significantly affecting the sign or relative size of the coefficients for the other independent variables.

The inclusion of the VSI (Lag –1) and MA (1) variables (required to address problems in serial correlations of the residuals) do not lend themselves to a simple economic interpretation. Only for the S&P 500 options, are both terms significantly important. Neither is important for the FTSE options and for the DAX and Nikkei markets, only one of the two terms is significant (although the lagged VSI is barely significant for the DAX).

6.2 BOND OPTIONS

For the Bond options markets, impacts of the independent variables are relatively consistent (Table 2). While the explanatory power of the models is slightly less than those observed for the Stock Index markets, the adjusted R-squared remains high (between 0.8457 for Gilt Options and 0.9243 for the US T-Bond Options). Furthermore, the Durbin-Watson statistics indicate no serious problems with serial correlations in the residuals.

As with the stock index options, for two of the markets (Bund and BTP futures), the pure first order strike price effect is insignificantly different from zero (β_1). The existence of a negative skew is due to market specific shocks for each of these markets (see β_5 and/or β_6). Market shocks also change the nature or the skewness effect for the other bond markets (Gilt and US T-Bond), including the 1987 Stock market crash. However, while market shocks tend to increase the degree of the negative skew for stock markets, the effect of the 1987 stock market crash was to make the US T-Bond skew more positive (although more negative for the Gilt options). The pure first order strike price effects for the Gilt and US T-Bond are of opposite signs and this effect is offset by the interaction with the level of the underlying (see β_8). For both markets, one interpretation of this interaction is that the degree of skewness is related to the level of the underlying asset. For the Gilts, negative skews become more prevalent when the level of the futures rise and for US T-Bonds, the level of negative skewness is reduced as the level of the futures price rises. For the bond markets, there is more consistency in the interaction of time and the level of the ATM implied volatility on the skewness (compared to the Stock Index Options markets). For all four markets (although insignificant for the US T-Bond), the higher the level of the expected variance, the more negative the degree of skewness becomes. This could provide evidence for a similar mechanism as the leverage effect identified for stock and stock index markets [See Christie (1982)]. What the nature of this mechanism is remains unknown.

Regarding time dependent effects, results suggest a similar mechanism exists for both Bond and Stock Index Options. The significant coefficients for both β_2 and β_3 suggest that the longer the term of the option, the more negatively skewed the pattern.

As with the stock index options, consistent second order strike price effects are found. Positive curvature is found (β_9) with similar time related dynamics. As with the stock index options, the curves become more extreme as the options expiration date is approached. Market shocks fail to have either a significant or consistent impact on the degree of curvature for Bond markets. For the two markets (Gilt and US T-Bonds) that were observed both pre and post the 1987 Stock market crash, this effect is slightly increased due to the crash. On the other hand, market specific shocks either increase or decrease the degree of curvature.

A significant negative relationship between the degree of the curvature and the level of the ATM implied volatility is also found (similar to that of stock index options). In addition, a similar negative relationship between the level of the underlying asset and the degree of curvature tends to occur (the exception is for the BTP options market). For these markets, it would appear that market agents lower their expectations about the level of excess kurtosis (for future market returns) when levels of expected market variance fall and prices of the underlying assets rise.

The final result which is consistent between bond and stock index options markets is the evidence of a significant third order strike price effect ($STRIKE^3$). For all eight markets, this effect was found to be positive and of a similar magnitude. One possible interpretation is that when market agents assess the excess kurtosis of future returns, they assign some degree of asymmetry to it.

For the remaining independent variables, no single variable is consistently significant for the four markets. For the US T-Bond options, many of the variables, which should be insignificant, are. This may suggest an error in the construction of variables may be present. However, alternative regression approaches have been able to eliminate these problems without substantial changes in the signs or levels of the other independent variables of interest. As with the stock index options, the inclusion of the VSI (Lag –1) and MA (1) variables are jointly only significant for the US T-Bond market. For the other markets, only the lagged VSI (Lag –1) variable is significant.

6.3 FOREIGN EXCHANGE OPTIONS

For the foreign exchange options markets, there is a further reduction in the explanatory power of the models. However, the worst R-squared statistic (of 0.8143 for the Swiss Franc) indicates that the model is still explaining the vast majority of the variance. As was previously reported for the Stock Index and Bond markets, the Durbin-Watson test statistics indicate no problems with serial correlation of residuals.

The coefficients for the first order strike price effects are divergent to the two previous asset classes and suggest alternative dynamics may be in effect. In Table 3, the pure first order strike price effect is significantly negative for three of the four markets, while this effect is time invariant. As was previously observed for US T-Bond options, this is offset by the positive relationship between the interaction of the level of the underlying futures price and the skewness. This suggests that when futures prices are low (high), the implied volatility pattern becomes more negatively (positively) skewed.

It appears that the skewness of the implied volatility pattern is not systematically affected by market specific shocks. The 1987 stock market crash has a small (but significantly) negative impact on the degree of skewness only for Deutsche Mark options. For both the Deutsche Mark and Japanese Yen markets, a negative relationship between the level of the ATM implied volatility and the degree of skewness is found. These results are similar to those found for the Stock Index and Bond option markets.

For the second order strike price effects, more consistency exists relative to the two previous asset classes. The pure curvature effect is consistently positive as is the first and second order impacts of TIME. As previously discussed, this result suggests that the curvature of the implied volatility patterns becomes more extreme as the options expiration date is approached. Market specific shocks do change the degree of curvature in the implied volatility pattern; tending to flatten the curve (including the 1987 stock market crash). Consistent with the two previous asset classes, a negative relationship is found between the curvature and the level of the ATM implied volatility. Finally, the level of the underlying futures does interact with the curvature of the implied volatility surface. Only for the Japanese Yen is this effect significant (and is slightly negative).

A significant third order strike price effect is observed for all four foreign exchange option markets. This effect is negative for all markets apart from the British Pound (which was only slightly positively significant). This is the opposite effect observed for stock index and bond options. Of the remaining independent variables, the only variable that is consistently significant across all the four markets is the ATMVOL variable. As the prior expectation is insignificance, the negative relationship could suggest some systematic error in the estimation of this variable has occurred. Alternative regression approaches serve to reduce the significance of this variable without substantially changing the results for the other independent variables. Finally, the variables included to address problems with serial correlations in residuals tend to be either insignificant (or low levels of significance).

6.4 FORWARD DEPOSIT OPTIONS

A review of the OLS Regression model for the deposit futures options markets, which appears in Table 4, seems to suggest these markets display similar dynamics to foreign exchange options markets. The levels of explanatory power of the model are similar and the Durbin-Watson statistics suggest the serial correlation problem has been addressed.

As with the currency options, the first order strike price effects are also fairly time invariant (apart from barely significant negative relationships between time and the skewness for the Euro Dollar and Euro Swiss options). When the pure first order strike price effect has a large positive or negative value, this is offset by a opposite relationship with the interaction between the level of the underlying futures price and the skewness. As previously suggested: when futures prices are at extreme levels, a skewed pattern occurs. Only for the Eurodollar and Euro D-mark markets, do specific shocks change the degree of skewness of the implied volatility patterns; however, this effect is neither systematic nor substantive. For only one market (Euro Sterling), was a significant relationship found between the level of the ATM implied volatility and the degree of skewness in the implied volatility pattern. Thus, it would appear that for deposit options, no systematic skew pattern exists (this is consistent with Figure 4). When such a pattern occurs this is related to extreme levels in the underlying forward interest rates.

For the second order strike price effects, results are consistent with those for the other markets considered. The implied volatility patterns of all four markets are positively curved and the first and second order impacts of TIME have a similar effect. As with the currency markets, market specific shocks change the degree of curvature in the implied volatility pattern: tending to flatten the curve (although the 1987 stock market crash slightly increased the curvature for the Euro Dollar options). Consistent across all asset classes, a significantly negative relationship is found between the degree of curvature and the level of the ATM implied volatility. In a similar manner to that observed for currency options, there tends to be a negative relationship between the level of the underlying futures and the degree of curvature of the implied volatility surface. For all four markets this effect is negative, however, for only two markets is this effect significant (Euro D-mark and Euro Swiss).

For the first time, the third order strike price variable, ($STRIKE^3$), is no longer significant for all four markets. Only for the Euro Swiss and Euro Dollar options is this significant and the signs of the regression coefficients are of opposite signs. Furthermore, the levels of the T-statistics are relatively low compared to the same T-statistics for this variable for markets in the other asset classes.

As the coefficients of the remaining variables vary across markets, we conclude that any errors in the measurement of our variables are neither systematic nor consistent across these four markets or between the four asset classes. Furthermore, there is no consistency in the significance the VSI (Lag –1) and MA (1) variables across these four markets (or across all sixteen markets, for that matter). Therefore, we conclude that the results presented here are not due to misspecification of the models or the estimation procedure and are robust.

7. THE PREDICTIVE POWER OF THE MODELS

A key concern in any modelling of this kind is that the high degree of explanatory power is due to over-fitting within a defined sample period. To address this issue two tests were performed. The first test was to rerun all the regressions with every contract included as a dummy variable. A contract is defined here as the individual expiration cycle. Given we have examined up to twelve years of options traded in the quarterly cycle, we have forty-eight (48) separate contracts. We found that few of the contract dummy variables were statistically significant and unsubstantial changes in the coefficients of the key variables of interest to this research were found. The second test entailed splitting the data set of available options prices into two sets. These periods were divided roughly into halves. Relying solely on data from the first half of the observations, we re-ran the regression [using the modified Newey-West (1987) estimator for the weighting covariance matrix] and used these results to predict the standardised implied volatilities in the second half of the available observations. The form of the regression model appears in equation 4.

$$VSI = \alpha + \beta \cdot VSI^{*} + \varepsilon \qquad \textbf{(4)}$$

Where VSI is the standardised implied volatility (smile index) outside of sample and VSI^* is the predicted standardised implied volatility (smile index) using the results from the regression (equation 3) using the first half of the data sample. Our criteria for gauging forecasting success outside of sample is the level of the adjusted R squared (to measure the efficiency of the model), and the coefficients of the regression equation (to assess whether the model is unbiased). The results of this test can be seen in Table 5 for all sixteen markets.

Regarding efficiency, if the degree of explanatory power of the models is retained compared to the first, second and overall periods, this indicates the estimation procedure is efficient. Typically, if a model is over-fitting within sample, the explanatory power will be lost outside of sample. In all cases this is not found and thus, we conclude that the results are not period specific.[21]

If the models are unbiased estimators, the intercept would be (insignificantly different from) zero and the slope coefficient would be (insignificantly different from) one. For seven of the sixteen markets, the model is an unbiased estimator of the relative implied volatilities in the out of sample period (DAX, Bund, US T-Bond, Deutsche Mark, Euro Dollar, Euro Sterling and Euro D-Mark). While for the other nine markets the model is a biased estimator, it could be that if the models were re-estimated through the sample (updating for the impacts of shocks that may have occurred), the estimators would then be unbiased. This remains for future research. However, it appears that these models are at the very least efficient estimators and could possibly be unbiased estimators (as the forecast horizon was shortened).

Under these assumptions, we conclude that regularities in implied volatility surfaces exist and are similar for markets in the same asset classes. This result is fairly time invariant. Furthermore, much regularity exists for the implied volatility surfaces of all the markets examined. These general results provide means to test alternative models, which could potentially explain why implied volatility surfaces exist. An extension to this research considers this problem and asks the question which possible models produce options prices that are consistent with the results presented here.

8. CONCLUSIONS AND IMPLICATIONS

Of considerable interest to both practitioners and academics is a rational explanation for the existence of implied volatility smiles. Prior to the development of such an explanation, it would be helpful if the empirical dynamics of implied volatility surfaces could be better understood. Dumas, Fleming and Whaley (1996, 1998) completed such research based upon the levels of implied volatilities and rejected the existence of a deterministic volatility function.

This research looks at implied volatility functions in a different light by separating out the impacts of implied volatility levels and concentrating solely on the relative shapes of implied volatility surfaces. Dumas, Fleming and Whaley (1996, 1998) may be correct that the levels of implied volatilities today (and across strikes) provides no meaningful information regarding future levels of implied volatility. However, this research demonstrates that this result might be due to a stochastic implied volatility process. Once levels are controlled for, regularities in relative surfaces are observed.

From an examination of implied volatility surfaces for sixteen options markets (representing a cross section of Stock Indices, Bonds, Currencies and Forward Deposits), we demonstrate that consistencies exist in the shapes of standardised surfaces for the options in the same asset class. A functional form was formulated to better understand the general behaviours. This functional form was evaluated using a modified weighted least squares regression, which used the Newey-West (1987) estimator for the weighting covariance matrix. Using this approach, vast majority (from 80-97%) of the variance in the implied volatility surfaces for the sixteen option markets was explained. Tests of the models outside of sample suggest the models are in all cases efficient estimators and in many cases, unbiased estimators of future relative volatility patterns. The consistencies between the models and the stability over time may suggest that market participants are using a similar algorithm over-time to adjust option prices away from Black-Scholes-Merton values and also may be using the same algorithm for different option markets.

The fact that all markets seem to rely on similar algorithm for adjusting option prices away from Black Scholes values has implications for the testing of the information content of implied volatility smiles. If smiles reflect market expectations of future asset price processes, then these shapes must vary as new information arrives in the marketplace. We find that standardised smiles do not vary substantially over time. Recently, Bates (1999) examined implicit distributions associated with options on the S&P 500 futures for a period post the 1987 crash and found that options prices did not adjust as extreme (negative) moves failed to occur. His work also suggests that the implied volatility smiles are stable over time and fail to incorporate new information.

The link between this research and Dumas, Fleming and Whaley (1996, 1998) is the separation of the implied volatility functional form into two processes. The first process describes the dynamics of the levels of implied volatility (and futures prices). In this research, the second process was examined: how the relative levels of implied volatilities vary across strike prices and time. Assuming the first process can be identified, a two-step implied volatility functional form could be determined. Then, it may make sense to re-examine the question asked by Dumas, Fleming and Whaley (1996,1998). It is left for future research to examine how such an implied volatility function would perform relative to the Black-Scholes model.

A more direct line of subsequent research would be to better understand the nature of this algorithm. In subsequent research, alternative models are examined that have been proposed to explain the existence of implied volatility surfaces. These models are compared for internal consistency with these results. Such models would include the Constant Elasticity of Variance model of Cox and Ross (1976), Jump diffusion models and the stochastic volatility models of Heston (1993), Barndorff-Nielsen (1997) and Barndorff-Nielsen and Shephard (1999). Correlated stochastic processes are also considered. Alternatively, market imperfections would be considered as the reason for the existence of implied volatility surfaces.

This research stands alone by formulating the correct questions to ask; by uncovering regularities for implied volatility surfaces, which seem to summarise the general phenomenon both within and between option markets on financial assets. This provides future researchers with benchmarks for the comparison of suitable models and insights into future models to be developed.

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FOOTNOTES:

[1] The existence of the strike price effect has been pointed out extensively in the literature. Early examples include: Black (1975), MacBeth and Merville (1979), Galai (1983,1987) and Rubinstein (1985). Recent examples include: Xu and Taylor (1993) and Heynen, Kemna and Vorst (1994).

[2] Dumas, Fleming and Whaley (1998) acknowledge that their research assumes a "null hypothesis" that the volatility is an exact function of asset price and time. They also recognise that volatility may be stochastic and given the difficulty in estimation of these processes and preference free option valuation, they suggest this for further research. This research eliminates this problem by indexing all volatilities to the level of the ATM volatility. Subsequent research will examine whether stochastic volatility models are sufficient to explain the relative shapes of implied volatility surfaces.

[3] For the DAX options and FTSE options, these were actually on the cash index. However, these products were European style options expiring on the same day as the Futures for these markets. Thus, these options can be considered as de facto options on futures.

[4] These contracts all represent 3 month offered deposit rates between Banks in London for the respective currencies. The Sterling contract is commonly referred to as Short Sterling.

[5] In total, the number of option prices examined for all sixteen markets was 1,862,473. Given that we also had the underlying futures prices for the same dates (and at the same time) as the options, we were able to assure that both time series were consistent to each other. From this analysis, we were able to clean both series and assure our analysis was minimally impacted by errors in data.

[6] The London International Financial Futures Exchange (LIFFE):Euro Sterling, Euro D-mark, Euro Swiss, BTPs, Bunds, Gilts and the FTSE 100. The Chicago Board of Trade (CBOT): US T-Bond Futures and Options. The Deutsche Terminbörse (DTB): DAX futures and options, The Chicago Mercantile Exchange (CME): Euro Dollar, S&P 500, Nikkei 225, Deutsche Mark, British Pound, Swiss Franc and Japanese Yen.

[7] In the instance that put and call options with the same time to expiration and same striking prices have different implied volatilities, this indicates that Put-Call Parity is violated and that an arbitrage opportunity may exist. In reality, it would most probably suggest that one of the option prices might be "old". From the previously quoted references, this would most probably be the in-the-money option. Given that liquidity problems should not exist when dealing in the underlying futures, it would be a simple matter to combine the out-of-the-money options with a position in the futures contract to create an in-the-money option with exactly the same implied volatility. It might be possible for markets where selling the underlying asset is prohibited, one would have to examine put and call smiles separately. However, the restriction of this research to options on actively traded futures contracts precludes this case and thus, the smiles we have estimated are not two branches glued together at the at-the-money level, but (by Put-Call parity) seamless.

[8] This manner of expressing the strike price is similar to the d_2 term that appears in the Black Scholes formula. It is common market practice in the currency options market to express strike prices in terms of the delta $[N(d_2)]$ and quote implied volatilities relative to this. This approximately expresses equation 1 as a probability.

[9] It is acknowledged that whenever some method of standardisation is employed, a loss of information (detail) results. However, given our objective is to compare smile behaviours both cross-sectionally and across time, we believe the loss of information by standardising is more that made up by the ability to compare smile dynamics within and between markets more directly. Furthermore, we will subsequently test for the importance of the levels of the underlying asset and the levels of the ATM volatility to assess what is lost in the standardisation process. It will be demonstrated that for many of the markets, the levels of the expected volatility and the underlying asset do impact the shape of the implied volatility smile. However, these effects are secondary to the more general relative strike price effects.

[10] The first approach used was to determine the quadratic functional form that fits the volatility smile. This used a quadratic approach suggested by Shimko [see Shimko (1991,1993)]. We found two major problems with this approach. The first is that for many days, we had barely enough degrees of freedom (options prices) to determine the quadratic form. Secondly, many of our markets (the US T-Bond market in particular) were not well described by a quadratic function.

[11] Data was restricted to weekly observations to reduce the size of the data set.

[12] Later in this paper, we will demonstrate that to correctly understand the characteristics of implied volatility surfaces a simple quadratic model of this form is inadequate. However, this goal here is to generate implied volatility surfaces which will provide qualitative insights into the nature of the complete model that will be developed later.

[13] The results of the regressions are available from the author by request.

[14] The nature of this expansion will embed the functional form of Dumas, Fleming and Whaley (1996) and subsequent analysis will assess if the addition of higher moments is warranted.

[15] Given that there are two time related interactions for the first order strike price, for the sake of consistency, we added another second order time interaction for the second order strike price effect. Thus, the final model is a mixture of a Taylor's series expansion to degree three and four.

[16] The logarithm, rather the absolute level, was used due to the wide discrepancies in the levels of the futures for the sixteen markets.

[17] Standardisation of variables is common in economic problems, when the objective is to remove impacts of scaling. As was discussed previously, we are not interested per se in the absolute level of the volatility or of the smile but of the relative relationships. This will allow for both inter-temporal comparisons within markets and allow comparisons between markets. The inclusion of the levels of the ATM volatility and the futures price will provide a check that important dynamics of the models have not been missed.

[18] The t-statistics for all the independent variables indicate whether the coefficient is statistically significantly different than zero. For the intercept, the t-statistic indicates whether the coefficient (alpha) is statistically significantly different than 100.

[19] Exceptions include the variables that include SHOCK1 for the S&P and FTSE and variables that include CRASH for the DAX and Nikkei. For the S&P and FTSE, the CRASH and SHOCK1 represent the same event. For the DAX and Nikkei, given that the available observations were only available after the CRASH, it makes no sense to include a variable with no variance.

[20] The first order effects of time have a straightforward economic interpretation. A negative relationship indicates that more (less) the time to expiration, the more (less) negatively skewed the implied volatility surface. The higher order time effects do not lend themselves to such an interpretation. They are included solely to assess if higher order time effects exist, which the results suggest they do.

[21] As an alternative to test for model stability, a Chow test was done to assess if structural breaks occurred in the models over time and whether the coefficients of the model estimated during the first period are the same in the second period. For seven of the sixteen markets, the Chow test is rejected that the models differ over the latter period. Results available from the Author upon request

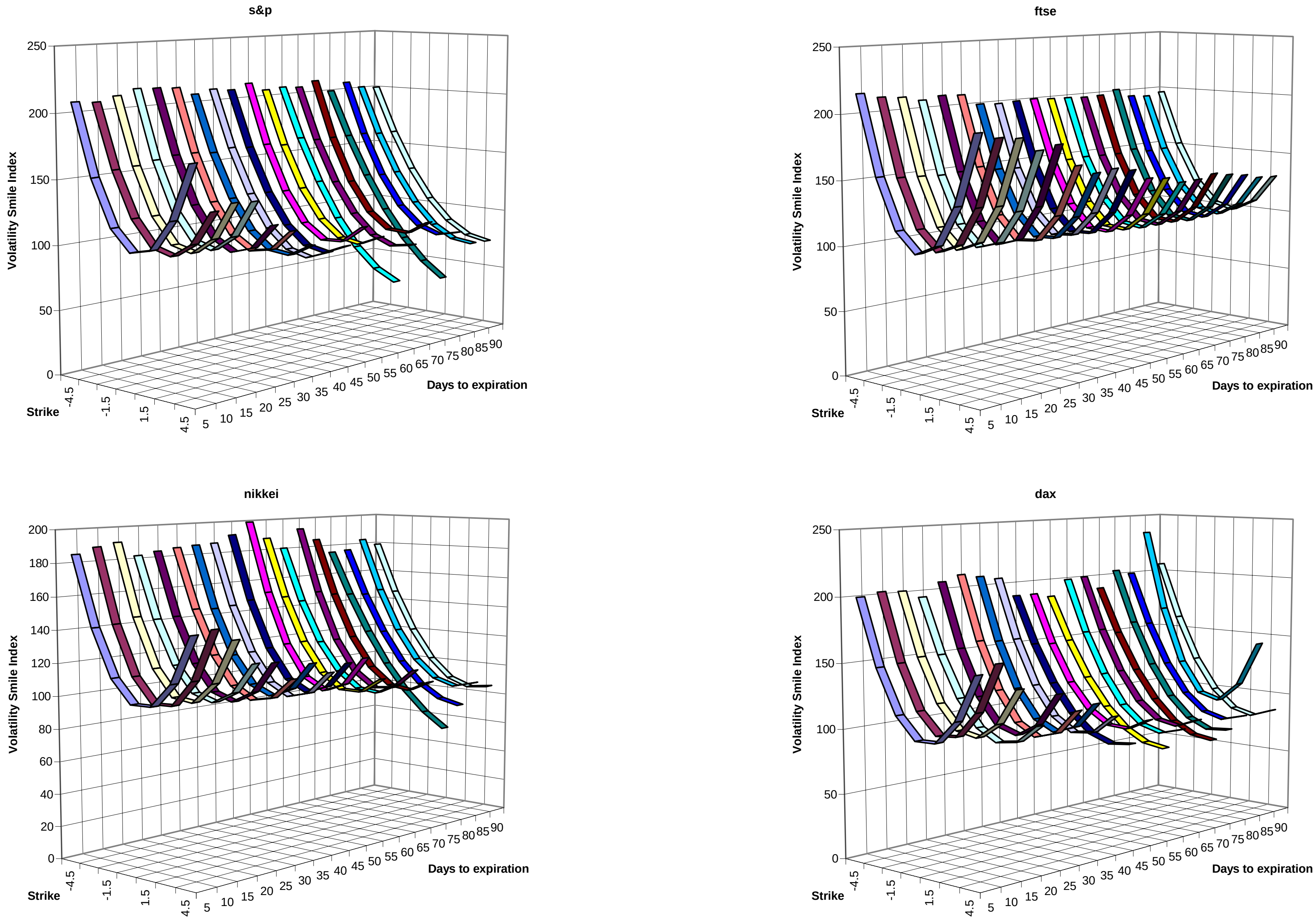

Figure 1 Standardized Volatility Smiles for Four Stock Index Options.

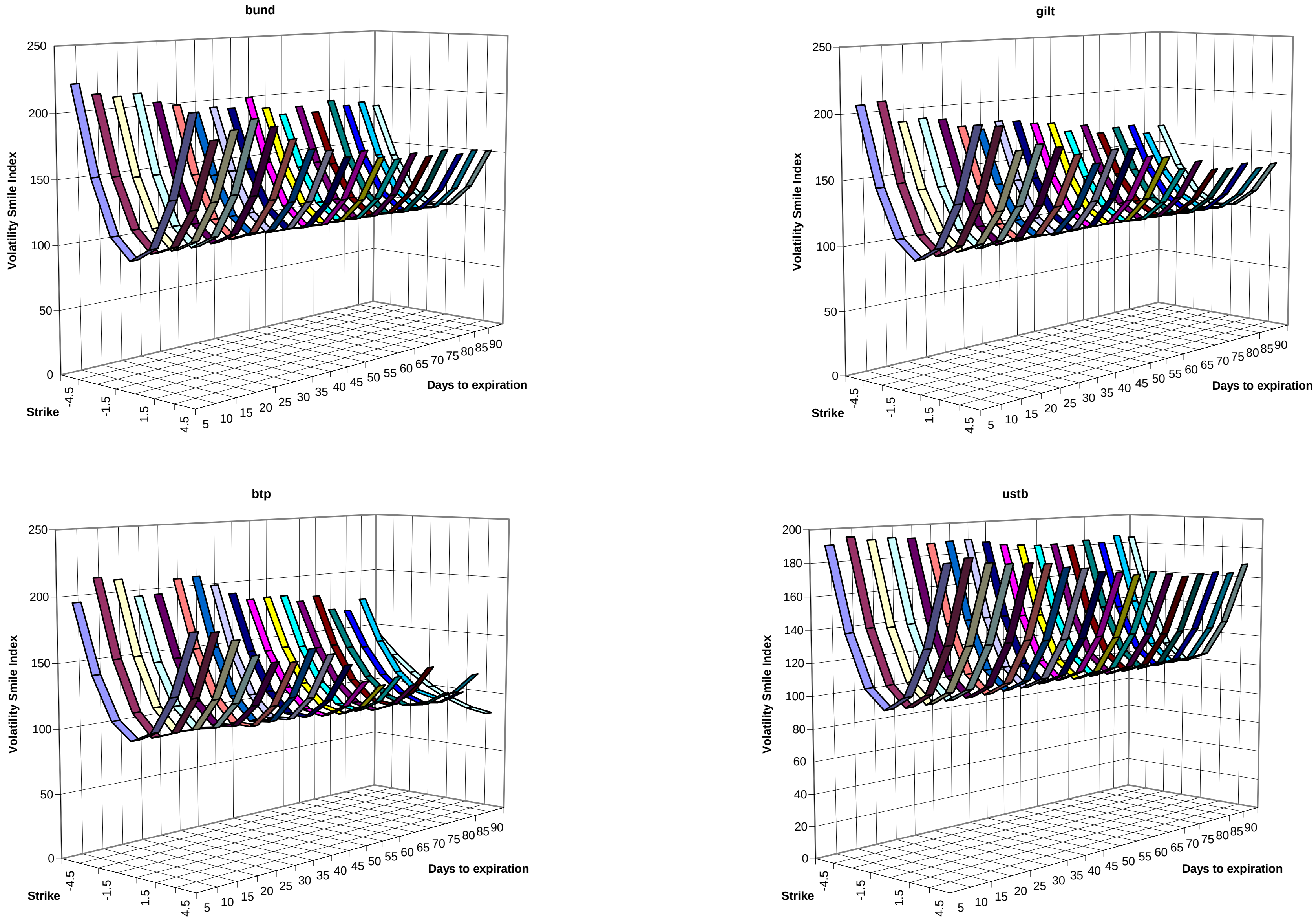

Figure 2 Standardized Volatility Smiles for Four Fixed Income Options.

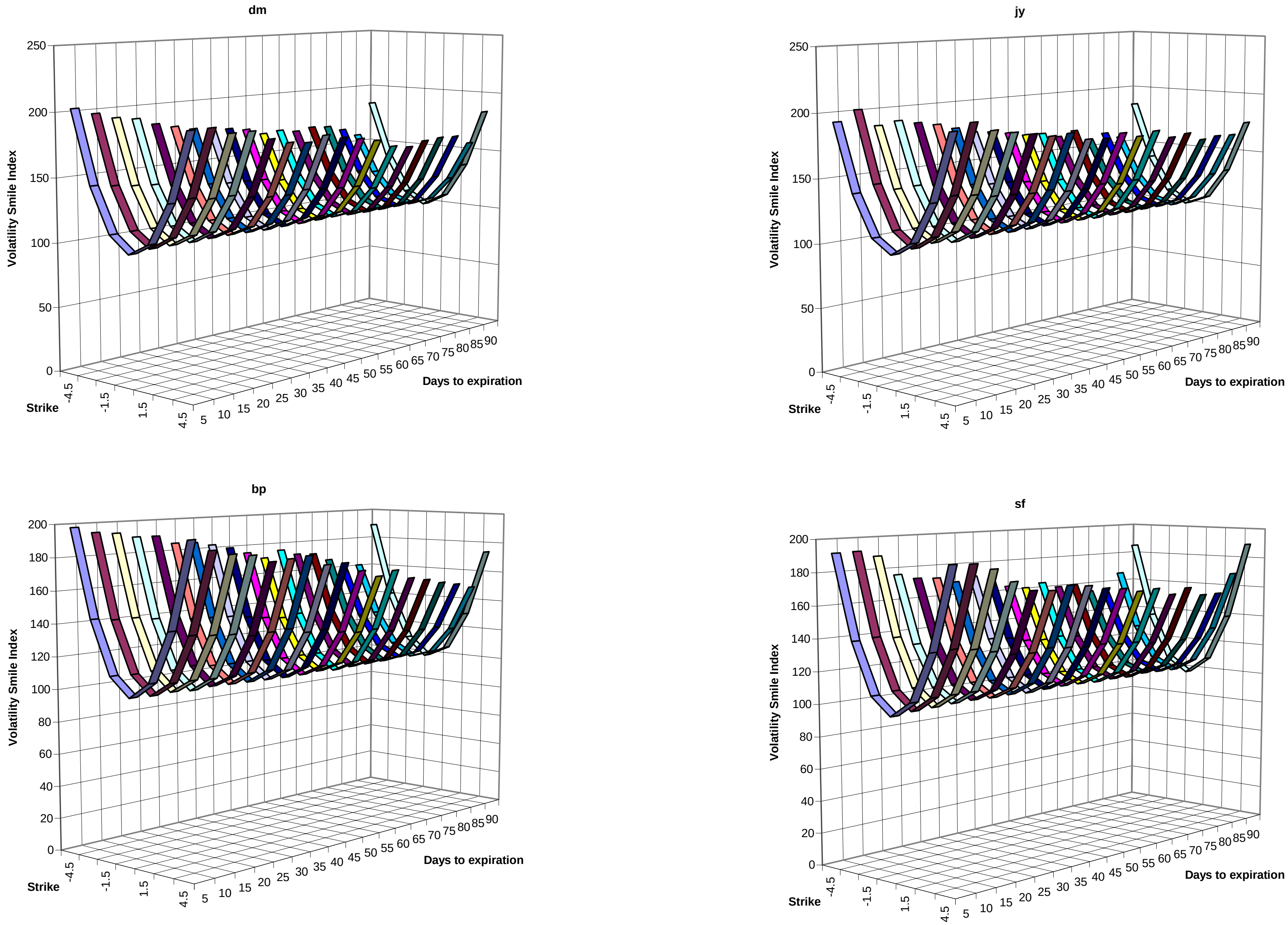

Figure 3 Standardized Volatility Smiles for Four Foreign Exchange Options.

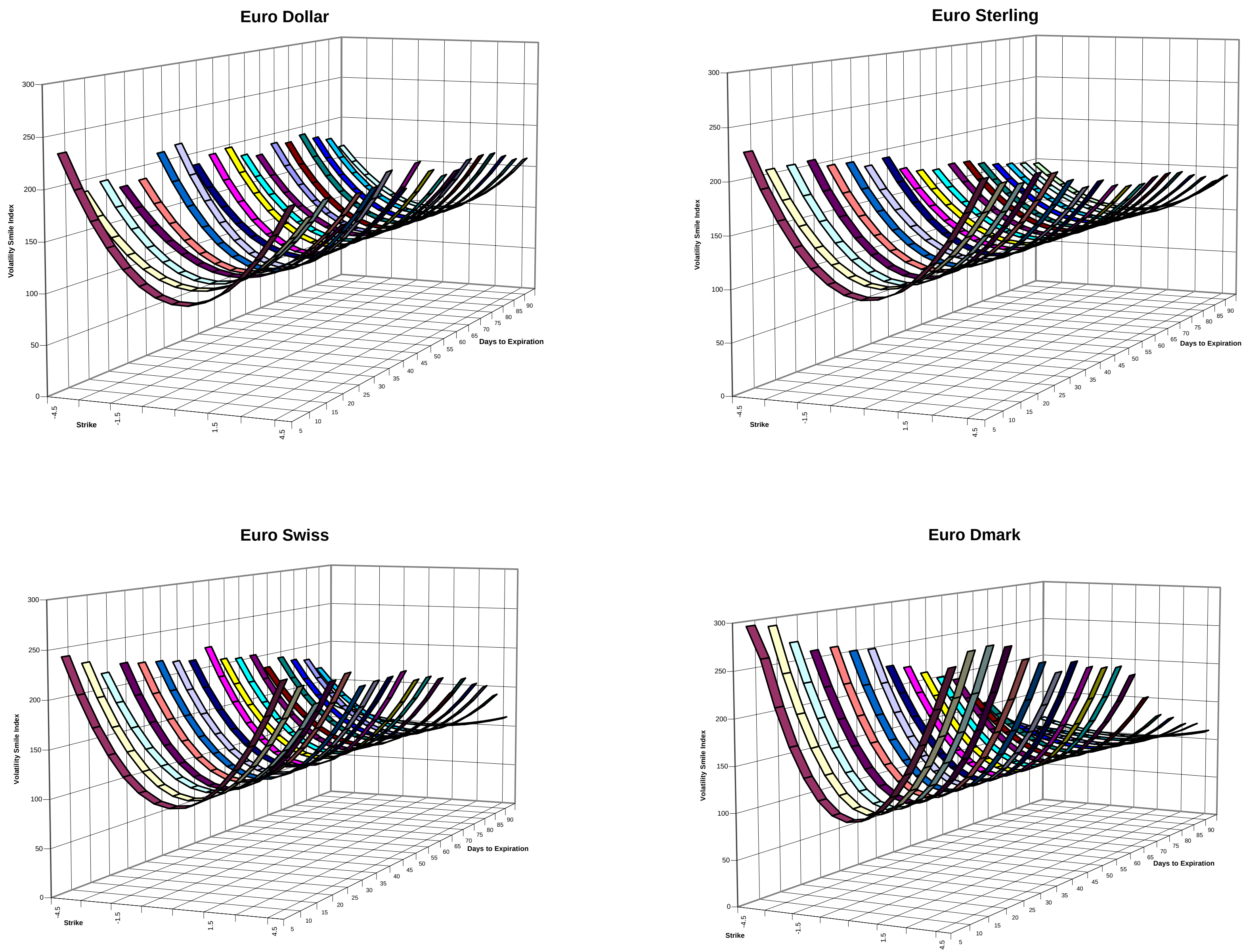

Figure 4 Standardized Volatility Smiles for Four Deposit Futures Options

FACTOR		S&P			FTSE			DAX			NIKKEI		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	**112.5529**	**3.8943**	**3.2234**	**144.4129**	**11.1383**	**3.9874**	**129.8403**	**11.8207**	**2.5244**	**66.7070**	**12.0736**	**-2.7575**
Strike	$\beta 1$	0.7758	3.2827	0.2363	-0.4717	8.4780	-0.0556	32.7169	17.8516	1.8327	-7.7881	13.0432	-0.5971
Strike*Time	$\beta 2$	**-60.6282**	**5.2257**	**-11.6019**	**-22.3234**	**6.7130**	**-3.3254**	**-54.1673**	**7.9548**	**-6.8094**	**-49.9658**	**7.1079**	**-7.0296**
Strike*Time^2	$\beta 3$	**145.6912**	**21.0371**	**6.9254**	51.4133	28.6409	1.7951	**109.2315**	**34.4809**	**3.1679**	**131.1086**	**27.1930**	**4.8214**
Strike*Crash	$\beta 4$	**-9.8164**	**1.1849**	**-8.2848**	**-5.1654**	**0.9317**	**-5.5444**	-.--	-.--	-.--	-.--	-.--	-.--
Strike*Shock1	$\beta 5$	-.--	-.--	-.--	-.--	-.--	-.--	0.9376	1.0075	0.9306	**3.1299**	**0.4703**	**6.6549**
Strike*Shock2	$\beta 6$	**-5.7732**	**0.4584**	**-12.5934**	-0.4780	0.7503	-0.6371	-0.1044	0.7926	-0.1317	-.--	-.--	-.--
Strike*ATMVol	$\beta 7$	**-12.6910**	**2.0084**	**-6.3188**	**9.2285**	**4.5735**	**2.0178**	2.5245	6.9852	0.3614	**-11.9768**	**3.3179**	**-3.6098**
Strike*Futures	$\beta 8$	0.5478	0.4992	1.0974	-0.2194	1.0955	-0.2003	**-5.2454**	**2.3364**	**-2.2451**	0.3171	1.5899	0.1995
Strike^2	$\beta 9$	**4.0899**	**1.3766**	**2.9711**	**11.0025**	**3.3226**	**3.3114**	**30.5738**	**7.2094**	**4.2408**	**27.8304**	**5.8869**	**4.7275**
Strike^2*Time	$\beta 10$	**-11.0515**	**2.1133**	**-5.2296**	**-5.2214**	**2.5553**	**-2.0434**	**-19.1005**	**4.5420**	**-4.2053**	**-16.4787**	**3.8459**	**-4.2848**
Strike^2*Time^2	$\beta 11$	**17.2993**	**8.6666**	**1.9961**	-6.6542	10.7172	-0.6209	39.4873	21.6987	1.8198	**43.5733**	**15.8246**	**2.7535**
Strike^2*Crash	$\beta 12$	0.1848	0.4914	0.3760	**-1.1158**	**0.4791**	**-2.3290**	-.--	-.--	-.--	-.--	-.--	-.--
Strike^2*Shock1	$\beta 13$	-.--	-.--	-.--	-.--	-.--	-.--	0.1566	0.6467	0.2421	0.2022	0.2403	0.8412
Strike^2*Shock2	$\beta 14$	**-0.7228**	**0.1741**	**-4.1525**	**-1.1992**	**0.3939**	**-3.0445**	-0.1715	0.4554	-0.3766	-.--	-.--	-.--
Strike^2*ATMVol	$\beta 15$	**-2.9292**	**0.8934**	**-3.2786**	**-9.8855**	**1.7387**	**-5.6856**	**-15.4169**	**2.8584**	**-5.3935**	**-8.2517**	**1.4342**	**-5.7537**
Strike^2*Futures	$\beta 16$	0.2752	0.2103	1.3086	-0.2862	0.4246	-0.6740	**-3.0408**	**0.9415**	**-3.2295**	**-2.8937**	**0.7249**	**-3.9919**
Strike^3	$\beta 17$	**0.7057**	**0.0284**	**24.8916**	**0.3712**	**0.0231**	**16.0859**	**0.3302**	**0.0423**	**7.8163**	**0.2875**	**0.0385**	**7.4582**
Crash	$\beta 18$	**2.3587**	**0.5038**	**4.6821**	**4.1239**	**0.8081**	**5.1032**	-.--	-.--	-.--	-.--	-.--	-.--
Shock1	$\beta 19$	-.--	-.--	-.--	-.--	-.--	-.--	0.0240	0.5375	0.0447	**0.9170**	**0.4123**	**2.2238**
Shock2	$\beta 20$	0.8997	0.4679	1.9227	0.9857	0.7698	1.2805	**1.1241**	**0.4337**	**2.5921**	-.--	-.--	-.--
ATMVol	$\beta 21$	**-13.6348**	**2.7373**	**-4.9812**	**-12.5904**	**4.8812**	**-2.5794**	-4.4018	5.0038	-0.8797	2.5642	3.1163	0.8228
Futures	$\beta 22$	**-2.7819**	**0.6335**	**-4.3915**	**-6.0349**	**1.4290**	**-4.2231**	**-4.5176**	**1.5681**	**-2.8810**	**3.4582**	**1.4963**	**2.3111**
Time	$\beta 23$	13.4882	17.3448	0.7777	-0.0743	24.2387	-0.0031	**70.9480**	**22.1621**	**3.2013**	44.7074	24.0370	1.8599
Time^2	$\beta 24$	-200.9370	135.1397	-1.4869	-307.0035	205.5521	-1.4936	**-571.7012**	**167.5899**	**-3.4113**	-174.9146	196.9818	-0.8880
Time^3	$\beta 25$	580.3829	321.6341	1.8045	**1405.4300**	**538.0266**	**2.6122**	**1337.3390**	**384.9428**	**3.4741**	100.6179	471.9337	0.2132
VSI (Lag -1)	$\beta 26$	**0.0374**	**0.0059**	**6.3617**	0.0064	0.0080	0.7981	0.0135	0.0086	1.5714	**0.0343**	**0.0124**	**2.7562**
MA (1)	$\beta 27$	**0.4377**	**0.0184**	**23.8351**	0.3405	0.3717	0.9161	**0.1662**	**0.0336**	**4.9466**	0.4282	0.3051	1.4034
			(Observations) (12387)			(Observations) (6980)			(Observations) (2768)			(Observations) (3525)	
		R-Squared	**0.9682**		R-Squared	**0.9051**		R-Squared	**0.9508**		R-Squared	**0.9331**	
		Durbin-Watson	1.7516		Durbin-Watson	1.8500		Durbin-Watson	1.9539		Durbin-Watson	1.6839	

Table 1 Newey-West Weighted Least Squares Results for Four Stock IndexOptions

FACTOR		**BUND**			**BTP**			**GILT**			**USTB**		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	90.1241	9.6324	-1.0253	93.4820	6.5113	-1.0010	**110.2127**	**4.9347**	**2.0696**	**164.2291**	**10.5907**	**6.0647**
Strike	$\beta1$	4.1695	10.1102	0.4124	-0.0069	6.2191	-0.0011	**14.5242**	**6.2741**	**2.3150**	**-21.1893**	**3.8365**	**-5.5230**
Strike*Time	$\beta2$	**-27.0100**	**4.8666**	**-5.5501**	**-35.9255**	**6.4038**	**-5.6100**	**-10.6773**	**4.1879**	**-2.5496**	**-6.5871**	**3.0859**	**-2.1346**
Strike*Time^2	$\beta3$	**90.5437**	**16.9592**	**5.3389**	**134.7295**	**22.6181**	**5.9567**	25.3171	14.5486	1.7402	7.2328	12.4586	0.5806
Strike*Crash	$\beta4$	-.--	-.--	-.--	-.--	-.--	-.--	**-1.8926**	**0.3837**	**-4.9321**	**1.2198**	**0.2285**	**5.3389**
Strike*Shock1	$\beta5$	**-1.8822**	**0.3334**	**-5.6459**	-0.2927	0.5469	-0.5351	-0.6180	0.3275	-1.8869	**-3.3954**	**0.5187**	**-6.5461**
Strike*Shock2	$\beta6$	**-2.0803**	**0.2568**	**-8.0997**	**-2.3887**	**0.2518**	**-9.4879**	**-3.5251**	**0.2257**	**-15.6212**	0.0064	0.1715	0.0376
Strike*ATMVol	$\beta7$	**-73.0321**	**6.8914**	**-10.5976**	**-38.7366**	**6.4640**	**-5.9926**	**-24.3095**	**4.4137**	**-5.5077**	-2.5196	3.3671	-0.7483
Strike*Futures	$\beta8$	0.1318	2.2145	0.0595	-0.0566	1.3433	-0.0421	**-2.3768**	**1.1924**	**-1.9933**	**4.4781**	**0.8098**	**5.5298**
Strike^2	$\beta9$	**23.3449**	**4.9258**	**4.7393**	**11.9812**	**3.2168**	**3.7246**	**18.5097**	**2.9154**	**6.3490**	**16.8941**	**1.3494**	**12.5196**
Strike^2*Time	$\beta10$	**-27.4792**	**2.8224**	**-9.7360**	**-5.1893**	**1.7108**	**-3.0333**	**-21.4837**	**2.4259**	**-8.8559**	**-11.9695**	**1.0283**	**-11.6406**
Strike^2*Time^2	$\beta11$	**71.7697**	**10.2548**	**6.9986**	24.1646	16.3645	-1.4766	**45.3785**	**9.2167**	**4.9235**	**29.1882**	**4.2724**	**6.8318**
Strike^2*Crash	$\beta12$	-.--	-.--	-.--	-.--	-.--	-.--	**0.7074**	**0.2178**	**3.2475**	**0.3074**	**0.0699**	**4.4000**
Strike^2*Shock1	$\beta13$	**1.0418**	**0.2142**	**4.8640**	-0.0617	0.2749	-0.2244	**1.2295**	**0.2767**	**4.4433**	-0.0934	0.1737	-0.5378
Strike^2*Shock2	$\beta14$	**-0.6297**	**0.1315**	**-4.7884**	0.2010	0.1072	1.8740	**-0.5041**	**0.1194**	**-4.2231**	-0.0649	0.0541	-1.1992
Strike^2*ATMVol	$\beta15$	**-12.3633**	**4.0766**	**-3.0327**	**-7.0910**	**3.2264**	**-2.1978**	**-25.7363**	**2.3080**	**-11.1512**	**-8.3491**	**0.9768**	**-8.5478**
Strike^2*Futures	$\beta16$	**-3.9625**	**1.0773**	**-3.6780**	**3.6632**	**0.7105**	**5.1555**	**-2.8461**	**0.5857**	**-4.8590**	**-2.4367**	**0.2920**	**-8.3450**
Strike^3	$\beta17$	**0.1192**	**0.0297**	**4.0085**	**0.3393**	**0.0280**	**12.1226**	**0.0996**	**0.0289**	**3.4430**	**0.1980**	**0.0107**	**18.5892**
Crash	$\beta18$	-.--	-.--	-.--	-.--	-.--	-.--	0.1391	0.4148	0.3355	-0.7634	0.7680	-0.9939
Shock1	$\beta19$	-0.1055	0.2959	-0.3564	-0.3266	0.5205	-0.6274	**-0.7773**	**0.3948**	**-1.9686**	**4.7248**	**0.9348**	**5.0545**
Shock2	$\beta20$	-0.3235	0.2334	-1.3859	**-0.6138**	**0.2360**	**-2.6012**	0.1447	0.1916	0.7553	**-1.4898**	**0.4927**	**-3.0237**
ATMVol	$\beta21$	**19.1036**	**6.5853**	**2.9009**	8.0921	5.3609	1.5095	5.4719	3.8139	1.4347	**-30.8738**	**8.7424**	**-3.5315**
Futures	$\beta22$	-1.6761	2.0563	-0.8151	-0.2517	1.3836	-0.1819	**-3.8454**	**0.9589**	**-4.0102**	**-14.1961**	**2.3370**	**-6.0745**
Time	$\beta23$	9.2376	17.7084	0.5217	-0.4178	19.5615	-0.0214	-12.4042	15.5380	-0.7983	**54.4361**	**19.0843**	**2.8524**
Time^2	$\beta24$	222.5889	139.2893	1.5980	57.2906	154.8615	0.3699	**305.6785**	**123.6076**	**2.4730**	**-417.0151**	**162.4247**	**-2.5674**
Time^3	$\beta25$	**-1060.6010**	**335.4081**	**-3.1621**	-233.6359	365.8936	-0.6385	**-972.1242**	**295.5201**	**-3.2895**	**1013.4470**	**417.9372**	**2.4249**
VSI (Lag -1)	$\beta26$	**0.1425**	**0.0176**	**8.1001**	**0.0648**	**0.0101**	**6.4046**	**0.0634**	**0.0096**	**6.5872**	**-0.0259**	**0.0080**	**-3.2395**
MA (1)	$\beta27$	0.2927	0.2171	1.3485	0.3465	0.2361	1.4677	0.2517	0.1891	1.3310	**0.3568**	**0.0195**	**18.3071**
			(Observations) **(8248)**			(Observations) **(8588)**			(Observations) **(9058)**			(Observations) **(9528)**	
		R-Squared	**0.8575**		R-Squared	**0.8946**		R-Squared	**0.8457**		R-Squared	**0.9243**	
		Durbin-Watson	1.8309		Durbin-Watson	1.8078		Durbin-Watson	1.8845		Durbin-Watson	1.7885	

Table 2 Newey-West Weighted Least Squares Results for Four Fixed Income Options

FACTOR		**D-MARK**			**POUND**			**YEN**			**S-FRANC**		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	**72.4670**	**4.9560**	**-5.5555**	**95.4712**	**1.3174**	**-3.4376**	94.6109	3.7485	-1.4377	**86.4974**	**3.0676**	**-4.4017**
Strike	$\beta1$	**-12.2603**	**5.0974**	**-2.4052**	1.6828	2.7099	0.6210	**-18.0848**	**3.4924**	**-5.1783**	**-13.7514**	**3.2133**	**-4.2796**
Strike*Time	$\beta2$	0.0099	3.3792	0.0029	2.8543	3.5800	0.7973	-2.7897	3.9805	-0.7008	2.8849	3.7101	0.7776
Strike*Time^2	$\beta3$	3.7369	12.9873	0.2877	-3.5049	13.4668	-0.2603	5.9217	15.5112	0.3818	-7.6842	14.9306	-0.5147
Strike*Crash	$\beta4$	**-0.8486**	**0.2560**	**-3.3147**	-0.4563	0.3146	-1.4504	-1.4999	0.8709	-1.7221	0.1470	0.8324	0.1766
Strike*Shock1	$\beta5$	**-3.1738**	**0.5966**	**-5.3199**	-1.8039	2.6943	-0.6695	1.3668	0.8234	1.6599	**-3.2878**	**0.4965**	**-6.6224**
Strike*Shock2	$\beta6$	0.2651	0.1689	1.5693	0.0834	0.1904	0.4379	**1.3197**	**0.2238**	**5.8976**	-0.6209	0.8415	-0.7378
Strike*ATMVol	$\beta7$	**-12.1557**	**3.0794**	**-3.9474**	4.0336	2.5539	1.5794	**-11.9640**	**3.4682**	**-3.4496**	-4.5003	2.8437	-1.5826
Strike*Futures	$\beta8$	**2.7393**	**0.8550**	**3.2040**	-2.0360	1.1933	-1.7063	**3.0821**	**0.5584**	**5.5197**	**2.7131**	**0.5208**	**5.2092**
Strike^2	$\beta9$	**9.8714**	**2.8745**	**3.4342**	**11.3579**	**2.0164**	**5.6328**	**11.1604**	**1.4097**	**7.9167**	**11.2045**	**1.8683**	**5.9972**
Strike^2*Time	$\beta10$	**-17.9669**	**1.6716**	**-10.7481**	**-14.8523**	**1.7838**	**-8.3264**	**-15.3218**	**1.9355**	**-7.9163**	**-20.9719**	**1.9401**	**-10.8099**
Strike^2*Time^2	$\beta11$	**44.5694**	**7.0987**	**6.2785**	**34.4158**	**8.1326**	**4.2318**	**35.5199**	**7.9802**	**4.4510**	**61.3308**	**8.3455**	**7.3489**
Strike^2*Crash	$\beta12$	0.2115	0.1097	1.9282	**-1.2826**	**0.1994**	**-6.4315**	**-0.7429**	**0.3345**	**-2.2212**	0.4690	0.2487	1.8859
Strike^2*Shock1	$\beta13$	**-1.0457**	**0.3070**	**-3.4064**	-3.4025	2.0096	-1.6931	**0.6886**	**0.3073**	**2.2407**	**-1.7776**	**0.2173**	**-8.1794**
Strike^2*Shock2	$\beta14$	-0.0696	0.0874	-0.7967	**0.3809**	**0.0953**	**3.9975**	**0.3517**	**0.0914**	**3.8465**	0.1932	0.2423	0.7977
Strike^2*ATMVol	$\beta15$	**-16.1779**	**1.5188**	**-10.6519**	**-20.2335**	**1.1213**	**-18.0451**	**-9.2916**	**1.5396**	**-6.0352**	**-16.1880**	**1.5785**	**-10.2552**
Strike^2*Futures	$\beta16$	-0.3591	0.4877	-0.7364	-0.1681	0.6069	-0.2769	**-0.7984**	**0.2270**	**-3.5172**	-0.5263	0.2965	-1.7750
Strike^3	$\beta17$	**-0.0840**	**0.0177**	**-4.7548**	**0.0301**	**0.0150**	**2.0033**	**-0.1617**	**0.0161**	**-10.0293**	**-0.0582**	**0.0181**	**-3.2203**
Crash	$\beta18$	-0.0735	0.2299	-0.3197	-0.0629	0.2575	-0.2443	**3.7814**	**0.7132**	**5.3021**	**1.2657**	**0.6408**	**1.9750**
Shock1	$\beta19$	**-1.3660**	**0.5604**	**-2.4374**	-0.3480	0.4673	-0.7448	**-3.9406**	**0.6837**	**-5.7638**	**-1.5283**	**0.5812**	**-2.6294**
Shock2	$\beta20$	**-1.1984**	**0.1601**	**-7.4845**	0.1498	0.1872	0.8003	**1.5237**	**0.2610**	**5.8381**	**-2.4258**	**0.6311**	**-3.8440**
ATMVol	$\beta21$	**-18.2090**	**3.4949**	**-5.2102**	**-7.8566**	**2.6705**	**-2.9420**	**-28.3911**	**4.6670**	**-6.0833**	**-33.6483**	**4.4511**	**-7.5595**
Futures	$\beta22$	**4.4408**	**0.8421**	**5.2734**	**3.7039**	**1.0443**	**3.5468**	0.5409	0.6279	0.8614	**2.1662**	**0.5119**	**4.2316**
Time	$\beta23$	24.7370	12.6599	1.9540	14.7252	15.1195	0.9739	22.3184	17.5110	1.2745	**46.0473**	**14.4805**	**3.1800**
Time^2	$\beta24$	-176.6624	103.5210	-1.7065	-159.3613	121.7005	-1.3095	-175.9706	137.7423	-1.2775	-229.3907	118.3906	-1.9376
Time^3	$\beta25$	347.4788	252.5897	1.3757	424.9866	289.9265	1.4658	319.9840	331.8644	0.9642	302.7454	292.3453	1.0356
VSI (Lag -1)	$\beta26$	**0.0309**	**0.0078**	**3.9708**	**0.0344**	**0.0122**	**2.8072**	**0.0497**	**0.0083**	**6.0226**	**0.0388**	**0.0119**	**3.2696**
MA (1)	$\beta27$	0.2094	0.1679	1.2470	**0.1578**	**0.0269**	**5.8774**	**0.2235**	**0.0201**	**11.1369**	0.1907	0.0185	10.3341
			(Observations) **(11079)**			(Observations) **(9190)**			(Observations) **(12998)**			(Observations) **(11834)**	
		R-Squared	**0.8679**		R-Squared	**0.8890**		R-Squared	**0.8785**		R-Squared	**0.8143**	
		Durbin-Watson	1.9307		Durbin-Watson	1.9635		Durbin-Watson	1.9289		Durbin-Watson	1.9664	

Table 3 Newey-West Weighted Least Squares Results for Four Foreign Exchange Options

FACTOR		**EuroDm**			**Sterling**			**Euro Swiss**			**Euro Dollar**		
		COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic	COEFFICIENT	Standard Error	T-Statistic
INTERCEPT	α	**83.2542**	**5.2596**	**-3.1839**	**72.9841**	**3.9575**	**-6.8266**	97.8610	3.6676	-0.5832	**96.8221**	**6.7225**	**-0.4727**
Strike	$\beta 1$	**15.7896**	**2.5422**	**6.2109**	0.5219	2.9857	0.1748	1.2848	2.3061	0.5571	**-21.5941**	**5.7825**	**-3.7344**
Strike*Time	$\beta 2$	-13.7193	12.8746	-1.0656	-9.9587	10.2413	-0.9724	**-23.1232**	**11.9202**	**-1.9398**	**-17.7000**	**6.5152**	**-2.7167**
Strike*Time^2	$\beta 3$	29.5152	40.7386	0.7245	35.0393	33.6628	1.0409	62.4600	40.4736	1.5432	**69.2100**	**23.4231**	**2.9548**
Strike*Crash	$\beta 4$	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	0.2459	0.8004	0.3073
Strike*Shock1	$\beta 5$	**-6.6494**	**0.9038**	**-7.3568**	-.--	-.--	-.--	-1.0652	0.8305	-1.2825	**-3.1015**	**0.7552**	**-4.1066**
Strike*Shock2	$\beta 6$	**1.7616**	**0.8478**	**2.0779**	-.--	-.--	-.--	-1.3984	1.0132	-1.3802	**-2.4742**	**0.5809**	**-4.2591**
Strike*ATMVol	$\beta 7$	-11.6524	6.7694	-1.7213	**-12.9478**	**3.6564**	**-3.5411**	-3.4349	2.6480	-1.2972	-3.1628	5.0025	-0.6322
Strike*Futures	$\beta 8$	**-5.9164**	**1.0163**	**-5.8216**	0.3498	1.2161	0.2876	-1.0008	0.7497	-1.3349	**4.1017**	**0.9568**	**4.2867**
Strike^2	$\beta 9$	**24.5748**	**2.0147**	**12.1978**	**14.7592**	**1.5607**	**9.4568**	**23.0928**	**1.0809**	**21.3640**	**6.1910**	**2.9800**	**2.0775**
Strike^2*Time	$\beta 10$	**-60.7235**	**7.3423**	**-8.2704**	**-34.8030**	**6.1817**	**-5.6300**	**-54.5427**	**4.8914**	**-11.1507**	**-8.1500**	**2.9380**	**-2.7740**
Strike^2*Time^2	$\beta 11$	**136.0443**	**24.0904**	**5.6472**	**57.2680**	**21.0374**	**2.7222**	**124.4244**	**16.7947**	**7.4085**	**30.4500**	**11.6875**	**2.6053**
Strike^2*Crash	$\beta 12$	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	**0.6162**	**0.3067**	**2.0091**
Strike^2*Shock1	$\beta 13$	**-2.0982**	**0.8074**	**-2.5985**	-.--	-.--	-.--	-0.5744	0.5726	-1.0031	**-2.0867**	**0.3187**	**-6.5472**
Strike^2*Shock2	$\beta 14$	1.0541	0.6143	1.7159	**1.6170**	**0.4446**	**3.6369**	**-2.5406**	**0.6287**	**-4.0408**	0.1289	0.2971	0.4338
Strike^2*ATMVol	$\beta 15$	**-22.3818**	**5.5830**	**-4.0089**	**-24.5796**	**2.1681**	**-11.3369**	**-15.3987**	**1.4198**	**-10.8454**	**-6.4303**	**2.6758**	**-2.4031**
Strike^2*Futures	$\beta 16$	**-3.9859**	**0.8164**	**-4.8822**	-1.0107	0.5883	-1.7182	**-4.5472**	**0.4217**	**-10.7835**	-0.1122	0.5378	-0.2087
Strike^3	$\beta 17$	-0.0152	0.1447	-0.1053	-0.0037	0.0693	-0.0530	**0.1918**	**0.0872**	**2.2004**	**-0.0375**	**0.0133**	**-2.8285**
Crash	$\beta 18$	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	-.--	**1.3918**	**0.6303**	**2.2083**
Shock1	$\beta 19$	**6.8496**	**1.7986**	**3.8082**	-.--	-.--	-.--	**-3.3670**	**1.1611**	**-2.8998**	**-3.1015**	**0.7552**	**-4.1066**
Shock2	$\beta 20$	**-3.7482**	**1.5513**	**-2.4161**	1.8905	1.0022	1.8864	1.3773	1.0910	1.2624	**-2.4742**	**0.5809**	**-4.2591**
ATMVol	$\beta 21$	**-37.5994**	**13.5661**	**-2.7716**	-0.4710	4.8882	-0.0963	-5.5316	3.7994	-1.4559	**-80.5095**	**7.7229**	**-10.4248**
Futures	$\beta 22$	-.--	-.--	-.--	-.--	-.--	-.--	-1.0516	1.1983	-0.8775	**2.9781**	**1.0658**	**2.7941**
Time	$\beta 23$	52.0547	47.0951	1.1053	12.2551	37.4148	0.3275	42.4584	54.6731	0.7766	**-45.6482**	**16.5462**	**-2.7588**
Time^2	$\beta 24$	-29.4479	359.5003	-0.0819	-1.0817	281.2288	-0.0038	0.0745	420.1949	0.0002	193.5206	141.0062	1.3724
Time^3	$\beta 25$	-356.9747	830.2662	-0.4300	-198.4319	649.7100	-0.3054	-386.5366	975.8212	-0.3961	-262.2998	363.0742	-0.7224
VSI (Lag -1)	$\beta 26$	**0.0774**	**0.0117**	**6.6183**	**0.1018**	**0.0115**	**8.8821**	0.0092	0.0152	0.6052	0.0131	0.0180	0.7255
MA (1)	$\beta 27$	**0.2945**	**0.0272**	**10.8419**	0.3541	0.4043	0.8757	0.2686	0.6143	0.4373	**0.1907**	**0.0250**	**7.6185**
			(Observations)			(Observations)			(Observations)			(Observations)	
			(3047)			**(5306)**			**(2523)**			**(5660)**	
		R-Squared	**0.8796**		R-Squared	**0.8118**		R-Squared	**0.8689**		R-Squared	**0.8006**	
		Durbin-Watson	1.8365		Durbin-Watson	1.8090		Durbin-Watson	1.9098		Durbin-Watson	1.9447	

Table 4 Newey-West Weighted Least Squares Results for Four Forward Deposit Options

Underlying Asset	Alpha	(Std Error)	(T-test)	Beta	(Std Error)	(T-test)	R Squared	Number of Observations
S&P 500 Futures	-6.4771	0.2826	-22.9192	1.0945	0.0024	39.1624	0.9709	6170
FTSE Futures	-2.6428	0.8724	-3.0295	1.0192	0.0075	2.5532	0.8797	3439
Nikkei 225 Futures	-3.0710	1.1890	-2.5829	1.0317	0.0111	2.8564	0.8907	1728
DAX Futures	-2.7356	1.4186	-1.9283	1.0195	0.0125	1.5535	0.8668	1399
Bund Futures	2.3793	1.4596	1.6301	0.9733	0.0133	-2.0089	0.8557	4248
BTP Futures	-2.7959	0.8734	-3.2010	1.0268	0.0082	3.2848	0.8484	4322
Gilt Futures	-8.1257	1.4601	-5.5653	1.0733	0.0136	5.3972	0.8148	4616
US T-Bond Futures	0.6199	0.5992	1.0344	0.9920	0.0048	-1.6652	0.8984	4859
Deutsche Mark /US $	0.5493	0.6637	-0.8276	0.9980	0.0061	-0.3336	0.8253	5694
British Pound /US $	-9.1584	0.6687	-13.6954	1.0949	0.0061	15.5463	0.8728	4685
Japanese Yen /US $	-17.0762	0.7984	-21.3870	1.1641	0.0073	22.6320	0.7955	6628
Swiss Franc /US $	-2.2100	0.7320	-3.0191	1.0102	0.0067	1.5157	0.7873	6063
Euro Dollar Futures	1.4041	1.0012	1.4024	0.9733	0.0109	-2.4471	0.8523	2423
Euro Sterling Futures	2.7785	1.9448	1.4287	0.9695	0.0186	-1.6401	0.7928	2604
Euro Dmark Futures	0.6241	0.9574	0.6518	0.9564	0.0076	-5.7252	0.8378	1557
Euro Swiss Futures	-1.9672	0.8008	-2.4565	1.0060	0.0066	0.9096	0.9031	1251

Table 5 Regression Results for the Predicted Standardised Implied Volatilities in the Second Half of the Options Data Set (Outside of Sample) using the Regression Model for the First Half of the Options Data Set.