Why we have always used the Black-Scholes-Merton option pricing formula

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Abstract

Derman and Taleb (*The Issusions of Dynamic Hedging*, 2005) uncover a seeming anomaly in option pricing theory which suggests that *static* hedging based on put-call parity provides sufficient theoretical support to justify risk-neutral option pricing. From this they suggest that *dynamic* hedging as a theoretical basis for the celebrated option pricing model of Black and Scholes (1973) and Merton (1973), while correct, is redundant [see also Haug and Taleb (*Why We Have Never Used the Black-Scholes-Merton Option Pricing Formula*, 2009)]. This paper examines the anomaly and finds that put-call parity does *not* provide a basis for risk-neutral option pricing.

Key words: Option pricing, put-call parity, dynamic hedging, static hedging, Black-Scholes-Merton

JEL codes: G12, G13

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Abstract

Derman and Taleb (*The Issusions of Dynamic Hedging*, 2005) uncover a seeming anomaly in option pricing theory which suggests that *static* hedging based on put-call parity provides sufficient theoretical support to justify risk-neutral option pricing. From this they suggest that *dynamic* hedging as a theoretical basis for the celebrated option pricing model of Black and Scholes (1973) and Merton (1973), while correct, is redundant [see also Haug and Taleb (*Why We Have Never Used the Black-Scholes-Merton Option Pricing Formula*, 2009)]. This paper examines the anomaly and finds that put-call parity does *not* provide a basis for risk-neutral option pricing.

1. Introduction

Since inception, the option pricing model of Black and Scholes (1973) and Merton (1973) has been aptly celebrated as a landmark development in financial economics. Their contribution is normally represented by their famous formula, though it is often thought to be misrepresented since similar formulas existed before 1973. In this view, the true achievement of Black-Scholes-Merton lies in their arbitrage-free, risk-neutral option pricing model based on dynamically hedging an option against its underlying security. However, even for algebraically identical formulas from the pre-Black-Scholes-Merton era the resemblance is superficial.[1] The Black-Scholes-Merton formula is distinguished by its origin within their dynamic hedging paradigm.

Recent debate criticizes their dynamic hedging paradigm. In particular, Derman and Taleb (2005) and Haug and Taleb (2009) make two distinct arguments in a critique of dynamic hedging à la Black-Scholes-Merton: 1) dynamic hedging is not feasible with sufficient precision to offer realistic empirical support for Black-Scholes-Merton option pricing theory, and 2) dynamic hedging is theoretically redundant since put-call parity already provides a theoretical basis for risk-neutral option pricing. The former argument is not addressed in this paper. There is already a large literature on this topic.[2] The latter argument regarding put-call parity as a theoretical basis for risk-neutral option pricing is the focal issue addressed here.

This paper proceeds as follows: we first review the argument in Derman and Taleb (2005) that put-call parity provides sufficient support to justify risk-neutral option pricing, thereby making dynamic hedging redundant. Next, we dissect the underlying structure of put-call parity to expose two constituent parity conditions that reveal a

[1] See, for example, Boness (1964), who *assumes for convenience* that investors in puts and calls are indifferent to risk.
[2] A partial sampling might include Bakshi, Cao and Chen (1997), Bossaerts and Hillion (1997), Boyle and Vorst (1992), Çetin et al. (2006), Constantinides and Zariphopoulou (1999), Dumas, Fleming and Whaley (1998), Figlewski (1989), Galai (1983), Leland (1985), and Li and Pearson (2007).

clear distinction between put-call parity and risk-neutral option pricing. In the following section we draw on Margrabe (1978) and generalize the discussion to include options to exchange assets. We subsequently discuss the relevance of the current discussion to formulas for the expected holding period return of an option derived in Rubinstein (1984). Finally, in the last section we state our conclusion.

2. Put-call parity and risk-neutral option pricing à la Derman and Taleb (2005)

Derman and Taleb (2005) suggest that a static hedging strategy based on put-call parity is sufficient to justify risk-neutral option pricing. Their argument challenges the importance of the contribution made by Black and Scholes (1973) and Merton (1973) and proceeds essentially as presented immediately below.

Let put-call parity be represented in the form shown in equation (1), where $C\left(S_0,K,T\right)$ and $P\left(S_0,K,T\right)$ denote standard European call and put options, respectively, with strike price K, time to expiration T, current price of a non-dividend paying security S_0, and where r denotes the riskless interest rate. [3]

$$C\left(S_0,K,T\right)-P\left(S_0,K,T\right)=S_0-e^{-rT}K \tag{1}$$

Actuarial price formulas for European call and put options on a non-dividend paying security are stated in equation (2), where g is the expected growth rate of the underlying security price and k is the rate used to discount expiration date payoffs for both options.

$$\begin{aligned} C\left(S_0,K,T\right)&=e^{-kT}\left(S_0e^{gT}N\left(d\right)-KN\left(d-\sigma\sqrt{T}\right)\right)\\ P\left(S_0,K,T\right)&=e^{-kT}\left(KN\left(-d+\sigma\sqrt{T}\right)-S_0e^{gT}N\left(-d\right)\right)\\ d&=\frac{\ln\left(S_0/K\right)+\left(g+\sigma^2/2\right)T}{\sigma\sqrt{T}} \end{aligned} \tag{2}$$

Substituting the actuarial call and put option prices from equation (2) into the put-call parity condition in equation (1) yields equation (3).

$$\begin{aligned} &S_0e^{(g-k)T}N\left(d\right)-e^{-kT}KN\left(d-\sigma\sqrt{T}\right)\\ &\qquad -e^{-kT}KN\left(-d+\sigma\sqrt{T}\right)+S_0e^{(g-k)T}N\left(-d\right)=S_0-e^{-rT}K \end{aligned} \tag{3}$$

Equation (3) reveals that the requirement of mutual consistency between the put-call parity condition in equation (1) and the call and put option price formulas in equation (2) dictates that both the discount rate k and the growth rate g be equal to the riskless rate r. With these equalities, i.e., $k=g=r$, the call and put prices in equation (2) are equivalent to the corresponding Black-Scholes-Merton formulas. On

[3] Stoll (1969) is a widely cited reference on put-call parity.

this basis, Derman and Taleb (2005) conclude that put-call parity is sufficient for risk-neutral option pricing. Their argument is remarkably simple. But is it correct?

Ruffino and Treussard (2006) opine otherwise. They suggest that a single non-stochastic discount rate both for both the call and the put is inconsistent with asset pricing theory. They also point out that allowing different discount factors for the call and the put does not alleviate the inconsistency with put call parity.[4] This point is acute, as is easily demonstrated. Let k_C and k_P denote separate discount rates for the call option and the put option. Insertion into equation (3) yields the expression shown immediately below, which reveals that allowing different put and call discount rates exacerbates the inconsistency with put-call parity.

$$S_0 e^{(g-k_C)T} N(d) - e^{-k_C T} KN\left(d - \sigma\sqrt{T}\right) - e^{-k_P T} KN\left(-d + \sigma\sqrt{T}\right) + S_0 e^{(g-k_P)T} N(-d) = S_0 - e^{-rT} K$$

The argument in Derman and Taleb (2005) depends crucially on restricting the call and put option price formulas in equation (2) to a single discount rate k. At first glance this seems natural, so accustomed are we to thinking within the risk-neutral paradigm. Of course, *after* publication of Black and Scholes (1973) and Merton (1973) the use of only a riskless discount rate to price options is a direct consequence of their dynamic hedging paradigm. But without the benefit of dynamic hedging à la Black-Scholes-Merton the assumption of a single discount rate is *ad hoc*.

To see why, consider the experiment of modifying the call and put option pricing formulas in equation (2) so as to contain two discount rates, h and k, where the discount rate k applies to the underlying security and the discount rate h applies to the strike price. In this case, we obtain the actuarial call and put option pricing formulas shown in equation (4).

$$\begin{aligned} C(S_0, K, T) &= S_0 e^{(g-k)T} N(d) - e^{-hT} KN\left(d - \sigma\sqrt{T}\right) \\ P(S_0, K, T) &= e^{-hT} KN\left(-d + \sigma\sqrt{T}\right) - S_0 e^{(g-k)T} N(-d) \\ d &= \frac{\ln(S_0/K) + \left(g + \sigma^2/2\right)T}{\sigma\sqrt{T}} \end{aligned} \quad (4)$$

Substituting the call and put option prices specified in equation (4) above into the put-call parity condition in equation (1) yields equation (5) immediately below.

[4] "In particular, the use of a non-stochastic discount rate, *k*, common to both the call and the put options is inconsistent with modern equilibrium capital asset pricing theory. Correspondingly, the use of valid discount factors—stochastic and different for the call and the put—would not allow Derman and Taleb to combine their 'actuarial' formulas, match the resulting expression with the forward price and still obtain the Black–Scholes formula. (Ruffino and Treussard, 2006, p.366)"

$$S_0 e^{(g-k)T} N(d) - e^{-hT} KN\left(d - \sigma\sqrt{T}\right) \\ - e^{-hT} KN\left(-d + \sigma\sqrt{T}\right) + S_0 e^{(g-k)T} N(-d) = S_0 - e^{-rT} K \quad (5)$$

Equation (5) reveals that the call and put option prices in equation (4) are only consistent with put-call parity in equation (1) when two conditions are jointly satisfied, 1) the discount rate for the underlying security is equal to its growth rate, i.e., $k = g$, and 2) the discount rate for the strike price is equal to the growth rate of a riskless discount bond, i.e., $h = r$. Importantly, it is *not* a requirement in equation (5) that the discount rate k for the underlying security be equal to the riskless rate r for consistency with put-call parity.

Derman and Taleb (2005) make the implicit assumption *a priori* that the discount rates h and k are one in the same. This leads them to conclude that put-call parity provides sufficient support for risk-neutral option pricing, thereby appearing to render dynamic hedging redundant. But with different discount rates h and k for the strike price and security price, respectively, their argument unravels.

3. Dissecting put-call parity

To see why, we must dissect put-call parity by drawing attention to the fact that standard European call and put options each contain two separate options. First, the call option $C(S_0, K, T)$ contains: 1) an asset-or-nothing binary call denoted by $AC(S_0, K, T)$, and 2) a strike-or-nothing binary call denoted by $KC(S_0, K, T)$. At option expiration, a long position in the asset-or-nothing binary call has the payoff $S_T \times I(S_T > K)$, where $I(x)$ is a zero-one indicator of the event x. Similarly, a short position in the strike-or-nothing binary call has the payoff $-K \times I(S_T > K)$. In combination, these long and short positions constitute a standard European call option as shown in equation (6).

$$C(S_0, K, T) = AC(S_0, K, T) - KC(S_0, K, T) \quad (6)$$

Second, the put option $P(S_0, K, T)$ contains: 1) a strike-or-nothing binary put denoted by $KP(S_0, K, T)$, and 2) an asset-or-nothing binary put denoted by $AP(S_0, K, T)$. At option expiration, a long position in the strike-or-nothing binary put has the payoff $K \times (1 - I(S_T > K))$ and a short position in the asset-or-nothing binary put has the payoff $-S_T \times (1 - I(S_T > K))$. Together, these long and short positions constitute a standard European put option as shown in equation (7).

$$P(S_0, K, T) = KP(S_0, K, T) - AP(S_0, K, T) \quad (7)$$

Substituting the asset-or-nothing and strike-or-nothing call and put options identified immediately above into the put-call parity condition in equation (1) reveals that put-call parity is actually a combination of two distinct parity conditions: 1) asset-or-nothing put-call parity, and 2) strike-or-nothing put-call parity. These two distinct parity conditions are stated in equation (8) below.

$$\begin{aligned} &AC\left(S_0, K, T\right) + AP\left(S_0, K, T\right) = S_0 \\ &KC\left(S_0, K, T\right) + KP\left(S_0, K, T\right) = e^{-rT} K \end{aligned} \qquad (8)$$

Expressed as an actuarial formula, the asset-or-nothing put-call parity condition is,

$$\begin{aligned} &S_0 e^{(g-k)T} N\left(d\right) + S_0 e^{(g-k)T} N\left(-d\right) = S_0 \\ &d = \frac{\ln\left(S_0 / K\right) + \left(g + \sigma^2 / 2\right)T}{\sigma\sqrt{T}} \end{aligned} \qquad (9)$$

The asset-or-nothing parity condition in equation (9) requires that the growth rate g be equal to the discount rate k, but is agnostic regarding the discount rate h.

Expressed as an actuarial formula, the strike-or-nothing put-call parity condition is,

$$\begin{aligned} &e^{-hT} KN\left(-d + \sigma\sqrt{T}\right) + e^{-hT} KN\left(d - \sigma\sqrt{T}\right) = e^{-rT} K \\ &d = \frac{\ln\left(S_0 / K\right) + \left(g + \sigma^2 / 2\right)T}{\sigma\sqrt{T}} \end{aligned} \qquad (10)$$

The strike-or-nothing parity condition in equation (10) requires that the discount rate h be equal to the riskless rate r, but is agnostic regarding the discount rate k.

The dissection of put-call parity above reveals that the discount rates k and h corresponding to the security price and strike price, respectively, are determined separately by the underlying parity conditions stated in equations (9) and (10). Nothing in the original put-call parity condition imposes equality on the discount rates k and h.

Call and put option prices satisfying the asset-or-nothing and strike-or-nothing parity conditions $k = g$ and $h = r$, respectively, are stated in equation (11) below. These are correct option prices *outside* the risk-neutral realm of Black-Scholes-Merton.

$$\begin{aligned} &C\left(S_0, K, T\right) = S_0 N\left(d\right) - e^{-rT} KN\left(d - \sigma\sqrt{T}\right) \\ &P\left(S_0, K, T\right) = e^{-rT} KN\left(-d + \sigma\sqrt{T}\right) - S_0 N\left(-d\right) \\ &d = \frac{\ln\left(S_0 / K\right) + \left(k + \sigma^2 / 2\right)T}{\sigma\sqrt{T}} \end{aligned} \qquad (11)$$

Equality of the discount rate k with the riskless rate r is not necessary for the call and put option prices in equation (11) to satisfy put-call parity. Risk-neutral option pricing à la Black-Scholes-Merton replaces the discount rate k with the riskless rate r. Put-call parity does not do this.

At this point it is worth noting that by setting $k = g$ in equation (2) above we obtain the option prices shown in equation (12) immediately below.

$$
\begin{aligned}
&C\left(S_0, K, T\right) = S_0 N\left(d\right) - e^{-kT} K N\left(d - \sigma\sqrt{T}\right) \\
&P\left(S_0, K, T\right) = e^{-kT} K N\left(-d + \sigma\sqrt{T}\right) - S_0 N\left(-d\right) \\
&\qquad d = \frac{\ln\left(S_0 / K\right) + \left(k + \sigma^2 / 2\right)T}{\sigma\sqrt{T}}
\end{aligned}
\tag{12}
$$

The call and put option prices in equation (12) are not legitimate outside the risk-neutral realm of Black-Scholes-Merton, where they are inconsistent with put-call parity. While they are rescued within the risk-neutral realm, they do not drag put-call parity with them. Put-call parity holds both inside and outside the risk-neutral realm.

4. Generalization to options to exchange assets

The discussion above gains strength and clarity by generalization to include options to exchange one asset for another. Standard call and put options are special cases of options with fixed strikes within a broader framework of options to exchange one asset for another. The discussion below draws heavily on Margrabe (1978).

Consider European call and put options to exchange the risky assets S and R at option expiration. The call grants the buyer the right to deliver asset R in exchange for asset S at contract expiration. The put grants the buyer the right to deliver asset S in exchange for asset R at contract expiration. A portfolio taking a long position in the call and a short position in the put will have the expiration date payoff indicated in equation (13).

$$
\max\left(0, S_T - R_T\right) - \max\left(0, R_T - S_T\right) = S_T - R_T \tag{13}
$$

Hence, the put-call parity condition for these call and put exchange options is expressed as shown in equation (14) immediately below, where $C\left(S_0, R_0, T\right)$ and $P\left(S_0, R_0, T\right)$ represent call and put option prices based on current security prices S_0, R_0 and time to option expiration T.

$$
C\left(S_0, R_0, T\right) - P\left(S_0, R_0, T\right) = S_0 - R_0 \tag{14}
$$

We shall assume that security prices S and R follow dynamic processes specified in equation (15), where g_S and g_R are security price growth rates, σ_S and σ_R are instantaneous return standard deviations, and Z_S and Z_R are Brownian motions with correlation parameter ρ_{RS}.

$$dS = \left(g_S - \sigma_S^2 / 2 \right) Sdt + \sigma_S S \sqrt{dt} dZ_S$$
$$dR = \left(g_R - \sigma_R^2 / 2 \right) Rdt + \sigma_R R \sqrt{dt} dZ_R \qquad (15)$$
$$E\left(dZ_S dZ_R \right) = \rho_{RS} dt$$

Given the discount rates k_S and k_R for securities S and R, respectively, the actuarial formulas for these call and put exchange options are stated in equation (16) below.

$$C\left(S_0, R_0, T \right) = S_0 e^{(g_S - k_S)T} N\left(d \right) - R_0 e^{(g_R - k_R)T} N\left(d - \hat{\sigma}\sqrt{T} \right)$$
$$P\left(S_0, R_0, T \right) = R_0 e^{(g_R - k_R)T} N\left(-d + \hat{\sigma}\sqrt{T} \right) - S_0 e^{(g_S - k_S)T} N\left(-d \right) \qquad (16)$$
$$d = \frac{\ln\left(S_0 / R_0 \right) + \left(g_S - g_R + \hat{\sigma}^2 / 2 \right) T}{\hat{\sigma}\sqrt{T}} \qquad \hat{\sigma}^2 = \sigma_S^2 + \sigma_S^2 - 2\rho_{RS}\sigma_S\sigma_R$$

Substituting the asset exchange option price formulas in equation (16) above into the put-call parity condition in equation (14) reveals two constituent parity conditions as shown in equation (17).

$$S_0 e^{(g_S - k_S)T} N\left(d \right) + S_0 e^{(g_S - k_S)T} N\left(-d \right) = S_0$$
$$R_0 e^{(g_R - k_R)T} N\left(-d + \hat{\sigma}\sqrt{T} \right) + R_0 e^{(g_R - k_R)T} N\left(d - \hat{\sigma}\sqrt{T} \right) = R_0 \qquad (17)$$

These constituent parity conditions require that $g_S = k_S$ and $g_R = k_R$, which in turn yields the refined parity conditions in equation (18) below.

$$S_0 N\left(d \right) + S_0 N\left(-d \right) = S_0$$
$$R_0 N\left(-d + \hat{\sigma}\sqrt{T} \right) + R_0 N\left(d - \hat{\sigma}\sqrt{T} \right) = R_0 \qquad (18)$$
$$d = \frac{\ln\left(S_0 / R_0 \right) + \left(k_S - k_R + \hat{\sigma} / 2 \right) T}{\hat{\sigma}\sqrt{T}}$$

Refined call and put exchange option price formulas satisfying the parity conditions in equation (18) above are stated in equation (19) immediately below. These are correct option prices outside the risk-neutral realm of Black-Scholes-Merton.

$$C\left(S_0, R_0, T \right) = S_0 N\left(d \right) - R_0 N\left(d - \hat{\sigma}\sqrt{T} \right)$$
$$P\left(S_0, R_0, T \right) = R_0 N\left(-d + \hat{\sigma}\sqrt{T} \right) - S_0 N\left(-d \right) \qquad (19)$$
$$d = \frac{\ln\left(S_0 / R_0 \right) + \left(k_S - k_R + \hat{\sigma} / 2 \right) T}{\hat{\sigma}\sqrt{T}}$$

Equality of the discount rates k_S and k_R is not necessary to satisfy put-call parity. Risk-neutral option pricing à la Black-Scholes-Merton replaces both discount rates k_S and k_R with the riskless rate r. Put-call parity does not do this.

As a final note to this section, again drawing on Margrabe (1978), letting $\sigma_R \to 0$ in equation (19) to yield $k_R = r$ and $R_0 = e^{-rT}K$ we obtain the pricing formulas for call and put options with fixed strikes given earlier in equation (11).

5. Expected holding period return of an option

Call and put option prices based on different discount rates for the underlying stock and strike prices have practical relevance when we are interested in option price behaviour outside the risk-neutral realm. Rubinstein (1984) provides an interesting application with formulas for expected future call and put option prices based on discount rates k and r for the stock and strike prices, respectively. In a notation adapted to the current text, these formulas are shown in equation (20) below, which state the expected future values of call and put options at time t given option expiration at time T, where $t < T$.

$$\begin{aligned} E(C_t) &= e^{kt}S_0N(d) - e^{-r(T-t)}KN\left(d - \sigma\sqrt{T}\right) \\ E(P_t) &= e^{-r(T-t)}KN\left(-d + \sigma\sqrt{T}\right) - e^{kt}S_0N(-d) \\ d &= \frac{\ln(S_0/K) + (k-r)t + \left(k + \sigma^2/2\right)T}{\sigma\sqrt{T}} \end{aligned} \tag{20}$$

These expected option prices based on the discount rates k and r for the underlying stock and strike prices, respectively, satisfy the following version of put-call parity:

$$E(C_t) - E(P_t) = e^{kt}S_t - e^{-r(T-t)}K \tag{21}$$

Rubinstein (1984) shows that the formulas for expected future option prices can provide expected future returns from investments in call and put options. In the current notation, the annualized expected holding period returns through time t for standard European call and put options are given in equation (21).

$$\begin{aligned} 1 + HPR_C &= \left(\frac{e^{kt}S_0N(d(k,r)) - e^{-r(T-t)}KN\left(d(k,r) - \sigma\sqrt{T}\right)}{S_0N(d(r,r)) - e^{rT}KN\left(d(r,r) - \sigma\sqrt{T}\right)}\right)^{1/t} \\ 1 + HPR_P &= \left(\frac{e^{-r(T-t)}KN\left(-d(k,r) + \sigma\sqrt{T}\right) - e^{kt}S_0N(-d(k,r))}{e^{-rT}KN\left(-d(r,r) + \sigma\sqrt{T}\right) - S_0N(-d(r,r))}\right)^{1/t} \\ d(b,a) &= \frac{\ln(S_0/K) + (b-a)t + \left(b + \sigma^2/2\right)T}{\sigma\sqrt{T}} \end{aligned} \tag{21}$$

It is straightforward to show that expected holding period returns for the asset-or-nothing call and put options, i.e., HPR_{AC}, HPR_{AP}, are:

$$1+HPR_{AC}=\left(\frac{e^{kt}N\left(d\left(k,r\right)\right)}{N\left(d\left(r,r\right)\right)}\right)^{1/t}\quad 1+HPR_{AP}=\left(\frac{e^{-r(T-t)}N\left(-d\left(k,r\right)+\sigma\sqrt{T}\right)}{N\left(-d\left(r,r\right)+\sigma\sqrt{T}\right)}\right)^{1/t}$$
$$d\left(b,a\right)=\frac{\ln\left(S_0/K\right)+\left(b-a\right)t+\left(b+\sigma^2/2\right)T}{\sigma\sqrt{T}} \tag{22}$$

Similarly, expected holding period returns for the strike-or-nothing call and put options, i.e., HPR_{KC}, HPR_{KP}, are:

$$1+HPR_{KC}=\left(\frac{e^{rt}N\left(d\left(k,r\right)-\sigma\sqrt{T}\right)}{N\left(d\left(r,r\right)-\sigma\sqrt{T}\right)}\right)^{1/t}$$
$$1+HPR_{KP}=\left(\frac{e^{rt}N\left(-d\left(k,r\right)+\sigma\sqrt{T}\right)}{N\left(-d\left(r,r\right)+\sigma\sqrt{T}\right)}\right)^{1/t} \tag{23}$$

6. Conclusion

Static hedging via put-call parity is not a sufficient theoretical basis for risk-neutral option pricing. We have always used the Black-Scholes-Merton option pricing formula because only the Black-Scholes-Merton formula originates within the dynamic hedging paradigm of risk-neutral option pricing.

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