Volatility Trade Design

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Abstract

Using data from the Eurodollar options on futures market, this paper examines six volatility trades: straddles, strangles, guts, butterflies, iron butterflies, and condors. We argue that straddles and strangles should have lower transaction costs than the other four strategies, and that (when constructed to be delta neutral) straddles, strangles, and guts should have higher vegas and gammas with a straddle's gamma and vega being the highest of the three. Consequently, we predict that in most situations volatility traders should prefer straddles and strangles to the other four strategies and that they should tend to favor straddles over strangles. Consistent with this we find that straddles account for 73.1% of all large volatility trades, strangles 20.8%, and butterflies 4.7% while the other three are rarely traded.

In general we find that most straddles and strangles are designed so that their delta is low and their gamma and vega are high (in absolute terms) but that they are not always constructed so that delta is minimized and vega and gamma maximized. Specifically, we find that most straddle traders choose the closest-to-the-money strike and that most strangle strikes are centered around the underlying asset price. While delta is low and gamma and vega high at these strikes, they may not be the delta minimizing and gamma/vega maximizing strikes. On the other hand, we find that when futures are added to a straddle position it is almost always in the ratio that reduces the delta of the position to zero and that the volatility trader's choice of whether to use a straddle or strangle depends on which can be designed with the lower delta.

There is little evidence that the shape of the smile impacts the strike price choices of straddle and strangle traders or that it impacts the straddle/strangle choice. We do find that the straddle/strangle choice depends on the time to expiration and whether the trader longs or shorts volatility.

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I. Introduction

By facilitating speculation on whether actual volatility will exceed or fall short of implied volatility and whether implied volatility will rise or fall, volatility trades, such as straddles, strangles, and butterflies, are important to the proper functioning of derivative markets. Such trades tend to equalize expected and implied volatility helping ensure that derivative securities are correctly priced. In a previous study, Chaput and Ederington (2002), we showed that much of the trading on option markets falls into the volatility trading category. For instance, we found that in the Eurodollar options market, straddle and strangle trades account for about 22.5% of all option trades of 100 contracts or larger.

Despite their importance to well functioning derivative markets and their popularity among option traders, volatility trades have received no attention in the financial research literature. While every derivatives textbook discusses such volatility trades as straddles, strangles, and butterflies and they are a staple of the practitioner literature, we have been unable to find a single research paper devoted to their design or trading.[1] To our knowledge, no one has documented which volatility trades are popular and which are not. As far as we can determine, no one has asked how volatility trades should be structured theoretically and no one has asked how they are structured in practice.

We seek to fill this gap by examining volatility trades on a major options market, the Eurodollar futures options market. First, we discuss attributes of six volatility trades or strategies: straddles, strangles, butterflies, condors, guts, and iron butterflies comparing them in terms of payoff patterns, delta, gamma, vega, and theta, price and expected payout, and transaction costs. We then document volatility trading activity in the Eurodollar market. As predicted by our analysis of the Greeks and transaction costs of the various strategies, we find that straddles are by far the most common volatility trade strategy followed by strangles while butterflies are a distant third. Condor, iron butterfly, and gut trades are quite rare. We then explore how the two most popular volatility trades, straddles and strangles, are structured (specifically the strike price choice(s)) and when traders choose a straddle over a strangle or vice versa. In general, we find that most (but not all) traders' choices conform to a model which presumes that volatility traders choose designs which maximize a combination's gamma and vega while minimizing its delta. We find little evidence that the design of straddle and strangle trades is influenced by the shape of the volatility smile. In other words there is no evidence that volatility traders choose designs which short options with high implied volatility and long those with low implied volatilities. This is consistent with the view that traders do not view the implied volatility differences as real but as resulting from calculations using an incorrect model.

The paper is organized as follows. In the following section, we discuss the six volatility trades outlining the advantages and disadvantages and developing predictions as to which strategies should be most attractive to traders. In III we describe our data. In IV we document which volatility strategies are most popular and their characteristics. The design of straddles is explored in section V and of strangles in section VI. Section VII considers the straddle/strangle choice decision and VIII concludes the paper.

II. Volatility Trades

In this paper we explore what might be called pure (or symmetric) volatility trades: straddles, strangles, butterflies, condors, iron butterflies, and guts. Descriptions of each are in Table 1. All have symmetric payout patterns which are illustrated in Figure 1 for positions which are long volatility. While ratio spreads are sometimes viewed as volatility trades, since they have a non-symmetric payoff pattern, and may be designed to exploit forecast directional changes in the underlying asset as well as forecast volatility, they are not considered here but are the subject of a separate paper.

Straddles, strangles and guts are usually referred to as "combinations" since one buys (or sells) both puts and calls while butterflies, iron butterflies, and condors are termed "spreads" since the trader buys some options and sells or writes others. However, for simplicity, here we often use the term "volatility spreads" (or "volatility trades") to refer to all six even though straddles and strangles are not technically spreads. We use the term "legs" to refer to the individual options making up a volatility spread.

We compare the six volatility trades on the following dimensions: payoff patterns (as illustrated in Figure 1), price, expected payout at expiration, their Greeks (specifically delta, gamma, vega, and theta) and transaction costs. This comparison then leads to hypotheses regarding the relative attractiveness of the various strategies to volatility traders. With the exception of our transaction cost comparison, most attributes of these strategies are well known in the literature but it is useful to review and compare them in order to form hypotheses regarding the relative attractiveness of the six strategies. It is assumed throughout that all have the same expiry.

II.1. Price and Payoff Characteristics

Payoffs at expiration for each of the six strategies as a function of the underlying asset price are illustrated in Figure 1 for positions which are long volatility. The payoff pattern for iron butterflies is identical to that for butterflies and that for guts is identical to strangles though the net price (or height of the curve) differs. As is well known and illustrated in Figure1, losses on bought straddles, strangles, and guts are bounded (to the paid price in the case of straddles and strangles) while potential profits are unbounded. If these combinations are sold, the opposite is true, that is potential losses are unbounded while potential profits are bounded. In the case of butterflies, iron flies, and condors both potential profits and losses are bounded.

In order to rank the six volatility spreads in terms of price (and expected payout) it is necessary to make some assumption about how they are constructed. For this, we assume that all are centered at roughly the underlying asset price so that (as shown below) the positions are approximately delta neutral. As shown below, this is both what we would normally expect and the case for most volatility trades. Considering first the three trades involving only two options, when their strikes surround the underlying asset price, guts have two legs in-the-money, straddles one, and strangles none. Hence, guts have the highest prices (and expected payouts) followed by straddles followed in turn by strangles. With butterflies, iron butterflies and condors , some options are bought and some sold so prices tend to be low. When centered at the money, their prices will be less than those of guts and straddles but whether they are more or less expensive than strangles depends on how each is constructed.

While a low price spread might appear desirable to a buyer and unattractive to a seller, this is balanced by the fact that the price of an option or spread is the discounted value of the (risk neutral) expected payout. Consequently, lower price spreads like butterflies also have lower expected payouts. While we expect these two attributes, price and expected payout, to balance out for traders in general, traders with a particularly high discount rate might prefer to buy (sell) combinations with a low (high) price while traders with a particularly low discount rate would prefer the opposite.

II.2. Spread Greeks

In all spreads and combinations (not just volatility trades), two or more options, or legs, are combined in order to exploit expected changes in one or more determinants of option value: the price of the underlying asset, its volatility, the time-to-expiration, or the interest rate while minimizing exposure to the other risks. Like its price, a spread or combination's "Greeks", delta, gamma, vega, theta, and rho,[2] are simple linear combinations of the derivatives for each of its legs. For instance, if a spread consists of M_1 contracts of option 1 and M_2 of option 2, the Greek of the spread, G_s, is $G_s = M_1 G_1 + M_2 G_2$ where G_1 and G_2 represent the Greek (delta, gamma, vega, theta, or rho) of the two legs and M is negative for a short position.

By definition, volatility traders seek to exploit either predicted changes in implied volatility and/or an anticipated difference between actual and implied volatility implying that they should seek positions with large vegas and/or gammas respectively. If they desire to minimize price risk, they should also seek positions which are delta neutral.[3] In all six volatility trades, the deltas of some legs have opposing signs so that the spreads can be made delta neutral. For instance, in a straddle, strangle, or gut, the trader buys (or sells) both a put and a call. Since a call has a positive delta and a put a negative delta, the deltas offset so the combination's delta is normally small. On the other hand, gamma, vega and theta are the same sign for both legs of straddles, strangles, and guts so these Greeks are normally sizable for the resulting combination. In the case of butterflies, iron butterflies, and condors, there is some offset in terms of all four Greeks.

In order to compare Greeks for the six volatility spreads, it is necessary to specify a pricing model. For this, we primarily use Black's options on futures model. Despite some shortcomings, this model is the one used by most traders in our market[4] and has the added advantage of tractability.

In Table 2, we present formulae for delta, gamma, vega, and theta for calls, puts, straddles, strangles (and guts), and butterflies (and iron butterflies) according to Black's model assuming volatility, σ, is the same at each strike (an assumption which is relaxed in section II.4). The expressions for condors, which are not shown, are simple extensions of the butterfly case. Note that both vega and gamma are proportional to the bracketed terms: $[2n(d)]$ for straddles, $[n(d_c)+n(d_p)]$ for strangles, and $[-n(d_1)+2n(d_2)-n(d_3)]$ for butterflies. In other words, both vega and gamma are proportional to $n(d)^{)} = \sum_i M_i n(d_i)$ where M_i is the number of options i bought or sold (in which case M is negative) and n() is the normal density function. This means that for a given expiry, gamma and vega are proportional. In other words, if switching from a strangle to a straddle or choosing different strike prices, raises gamma X%, it also raises vega X%. Consequently, we use $n(d)^{)}$ as a measure of both vega and gamma for strategies with the same expiry. In general, theta is approximately proportional to $n(d)^{)} = \sum_i M_i n(d_i)$ as well since the rP term is normally small.[5]

Among the six volatility spreads, the one with the largest Black values for gamma and vega is a delta-neutral straddle. As shown in Table 2, a straddle's Black delta is zero when N(d)= .5, or when d=0. Since $d = [\ln(F/X) + (.5\sigma^2 t)]/\sigma\sqrt{t}$, d=0 if the strike price $X = Fe^{.5\sigma^2 t}$ where F is the underlying futures price, σ is the instantaneous volatility and t is the time-to-expiration. We label this delta neutral strike F*. For short time-to-expiration options, the exponential term is small so F* is just slightly above the current futures price, F. For instance, if F=6.500 (its approximate mean in our sample),[6] σ =.16 (its approximate median in our sample) and t=.333 years (four months), delta is zero at the strike F* = 6.528. At a strike of F*, gamma and vega are also maximized since the normal density n(d) reaches its maximum of .39894 when d=0. So for a delta neutral straddle, $n(d)' = 2n(d) = \sum_i M_i n(d_i) = .7979$. For strangles, at least one strike must be different from F* so $d \neq 0$, and $n(d)' = \sum_i M_i n(d_i) < .7979$. For butterflies (and condors), some of the M_i are negative so again $\sum_i M_i n(d_i) < .7979$. Consequently, if a trader's goals are to maximize the position's Black gamma and vega while minimizing delta, she should choose a straddle with a strike equal to F*.

In order to illustrate how the Greeks differ among the six volatility trades, in Table 3, we report estimated Greeks (along with the net price and expected payout at expiration) according to both the Black model (Panel A) and the Barone-Adesi-Whaley (hereafter BW) American options model (Panel B) for the six volatility trades for representative market conditions. All are constructed for Eurodollar options assuming the underlying LIBOR rate is 6.465% so that F*= 6.50% (the approximate mean LIBOR rate in our sample), the volatility is 16% (again the approximate sample mean), and time-to-expiration is 5 months. All trades are constructed so that the mean strike equals 6.50%, i.e., they are centered close-to-the-money so that delta is approximately zero. For all except straddles and condors, we report two sets of results: one where the gap between the strikes is 25 basis points (the minimum for five month options in our sample) and one where it is 50bp. As expected, gamma and vega are highest for straddles and large for strangles and guts but much smaller for butterflies, iron flies, and condors. Indeed, the latter three appear to be very weak volatility plays. Note that the differences between the Black and BW Greeks are fairly small - except possibly for the deep-in-the-money guts.

In summary, the volatility spread with the highest Black model values for gamma and vega is a straddle at a strike price of $F^* = Fe^{.5\sigma^2 t}$. Serendipitously, this strike also results in a delta neutral position in the Black model. If centered at F* and constructed with a small gap between the strikes, strangles and guts also have relatively high gamma/vega values and are delta neutral. Generally, gamma and vega are considerably lower for butterflies, iron flies, and condors. If therefore the objective is to maximize gamma and vega while minimizing delta, we would expect straddles to be normally the first choice followed by strangles and/or guts.

II.3. Transaction Costs

We also expect a volatility traders's spread choice to depend on transactions costs consisting of bid/ask spreads and brokerage fees. We hypothesize that these should partially depend on the number of different options making up the combination. If a trader forms a straddle, strangle, or gut, she faces transaction costs on two options: the call and the put.[7] If she forms a butterfly, she faces transaction costs on three different options while iron flies and condors involve four. Consequently, transaction costs should be lowest for straddles, strangles, and guts and highest for iron flies and condors with butterflies in between.

While straddles, strangles, and guts all involve two options, their transaction costs may differ if some options are more heavily traded than others. In our market, trading is heavier in at-the-money and out-of-the-money options than in in-the-money options. For the same strikes, guts and strangles share identical payoff patterns and Greeks but guts have at least one (and normally both) legs in-the-money while at least one (and normally both) of an equivalent strangle's legs are out-of-the money. Consequently, we expect transaction costs to be lower on strangles than on guts which normally involve less liquid options.

In addition to the costs of constructing a spread position, if the positions are held to maturity, there are costs associated with exercising in-the-money options. When an option is exercised a brokerage fee is normally levied as if the option was sold. On exercise the long receives a long position in the underlying futures and the short a short position. When the futures position is closed, a further brokerage fee may be levied.[8] With guts, at least one (and maybe both) options must finish in-the-money; with straddles exactly one will finish in-the-money; and with strangles at most one option will finish in-the-money. Consequently expected exercise costs should be highest for guts and lowest for strangles with straddles in between.

II.4. Volatility Differences and Other Factors.

Our analysis of the Greeks and transaction costs of the volatility spreads indicates that most volatility traders should generally prefer straddles and strangles, and especially straddles, to the other volatility spreads. However, the other combinations might appeal to particular traders or be more attractive at particular times for other reasons. First, in the equations in Table 2, volatility, σ, is assumed to be the same at all strikes. As is well known, implied volatilities normally differ across strikes in a smile or smirk pattern. One theory of the smile is that it arises because the implied volatilities are being calculated using the wrong model, i.e., Black-Scholes. According to this view, if the implied volatilities were calculated using the correct model (often a model assuming non-lognormal returns or stochastic volatility), the implied volatilities would be the same. If traders concur with this view, we would expect no trading based on the apparent Black volatility differences. On the other hand, if traders view the Black volatilities as correct, so that they reflect options which are relatively over or under-priced, they may prefer to sell strikes with high implied volatilities and buy those with low volatilities leading to different combination choices. For instance, if the smile is U shaped with implied volatilities lowest for at-the-money strikes and higher for in-the-money and out-of-the-money strikes, they may prefer to buy at-the-money straddles but to sell strangles constructed with away-from-the-money strikes. Alternatively, they may prefer short butterflies, iron flies, or condors in which they buy the at-the-money strikes and sell the out-of-the-money strikes.

Second, because options are traded at only a few strikes, at times traders may choose strategies which would be their second choice if all strikes were available. For instance, in the Eurodollar options market, the available strikes are usually in increments of 25 basis points for expiries exceeding three months. If the underlying futures is about midway between two strikes, then (as we show in section VII below), a straddle cannot be made delta neutral while a strangle can. Consequently, a trader who would prefer a straddle to a strangle if all strikes were available, might choose a strangle instead. We find evidence of this in section VII.

Third, because of a particularly high or low discount rate, an individual trader might prefer combinations with very low net prices (and expected payouts), like butterflies, or very high, like guts. Fourth, due to risk preferences or other reasons, an individual trader may prefer one of the four payoff patterns in Figure 1, such as the bounded property of butterflies and condors. Finally, we would note that butterfly and condor trades can be used to exploit apparent mispricings - not just to bet on changes in actual or implied volatility. If the option at strike Y is viewed as overpriced relative to the strikes immediately above, X, and below, Z, then a trader may wish to construct a butterfly in which he shorts Y and longs X and Z expecting the prices to move back in line. In other words, while we have analyzed butterflies, iron butterflies, and condors as volatility trades, that is not their only possible role.

II.5. Summary

In summary, based on Greeks and transaction costs, butterflies, iron flies, and condors would appear to be relatively unattractive volatility trades in general since they have relatively low gammas and vegas and high transaction costs. While strangles and guts have identical Greeks and payoff patterns when constructed with the same strikes, guts probably entail higher expected transaction costs and involve less liquid options so would appear to be normally dominated by strangles. Consequently, in terms of Greeks and transaction costs, the two most attractive combinations would appear to be straddles and strangles. If both are constructed so as to be delta neutral, then straddles have somewhat higher gammas and vegas so appear preferable on this score.

This ranking assumes volatility traders seek to maximize gamma and vega and minimize delta and transaction costs and that, when correctly calculated, implied volatilities do not differ across strike prices. If traders are seeking to exploit apparent mispricings, if they have particularly high or low discount rates or if they have strong risk preferences, then they might prefer one of the other strategies: butterflies, iron flies, or condors.

III. Eurodollar Options Data

III.1. Data

As explained in Chaput and Ederington (2002), public options trading data does not identify spread and combination trades. However, data on large option trades in the Chicago Mercantile Exchange's market for Options on Eurodollar Futures (the world's most heavily traded short-term interest rate options market) with these identifiers was generously provided to us by Bear Brokerage.[9] Bear Brokerage regularly stations an observer at the periphery of the Eurodollar option and futures pits with instructions to record all options trades of 100 contracts or larger.[10] For each large trade, this observer records (1) the net price, (2) the clearing member initiating the trade, (3) the trade type, e.g., naked call, straddle, vertical spread, etc., (4) a buy/sell indicator, (5) the strike price and expiration month of each leg of the trade, and (6) the number of contracts for each leg. If a futures trade is part of the order, he also records the expiration month, number, and price of the futures contracts. Note that only the net price of the spread or combination, not separate prices for each leg, is normally observed and recorded.[11] The large trades recorded on the Bear Brokerage sheets account for approximately 65.8% of the options traded on the observed days.

We only observe spreads and combinations which are ordered as such. If an off-the-floor trader places two separate orders, one for 200 calls and another for 200 puts with the same strike and expiry, our records show two separate naked trades, not a straddle while, if he places a single order for 200 straddles, we observe a straddle. Consequently, our data may understate the full extent of volatility spread trading. However, if a trader splits his order, he cannot control execution risk. For example, if he orders 200 straddles, he can set a net price limit of 10 basis points. He cannot do this if he splits the order and, if he sets limits on each leg, one leg may wind up being executed without the other. Consequently, the traders to whom we have talked think the data capture almost all spread and combination trades.

Bear Brokerage provided us with data for large orders on 385 of 459 trading days during three periods: (1) May 12, 1994 through May 18, 1995, (2) April 19 through September 21, 1999 and (3) March 17 through July 31, 2000. Data for the other 74 days during these periods was either not collected due to vacations, illness, or reassignment or the records were not kept. After applying several screens to our data to remove trades solely between floor traders and likely recording errors as described in Chaput and Ederington (2002), the resulting data set consists of 13,597 large trades on 385 days of which 3256 represent one of our six volatility trades. We also obtained data on daily option and futures prices: open, high, low, and settlement along with implied volatilities from the Futures Industry Institute.

III.2. Volatility Patterns.

As discussed in section II.4 above, if traders view implied volatility differences as genuine, i.e., not due to calculation by the wrong model, then volatility traders may prefer to short options with relatively high implied volatilities and long those with low implied volatilities. Consequently, we explore below whether volatility spread design depend on the shape of the smile. Accordingly, in Table 4 and Figure 2 we document the normal smile pattern in implied volatilities in the Eurodollar options market over our data period.[12] For each option j on every day t, we obtain the implied standard deviation, $ISD_{j,t}$, based on day t's settlement prices as calculated by the CME and calculate the relative percentage "moneyness" of option j's strike price measured as $(X_{j,t}/F_t)-1$ where $X_{j,t}$ is option j's strike price and F_t is the underlying futures price on day t. This is done for two different expiries: options maturing in two to six weeks and options maturing in 13 to 26 weeks. Time series means of both ISD and (X/F)-1 are reported in Table 4 and the former is graphed against the latter in Figure 2. The following nomenclature is used in Table 4 to identify calls and puts and strike price groups j. The first letter, "C" or "P," indicates **c**all or **p**ut, the second, "I" or "O", indicates whether the option is **i**n or **o**ut of the money, and the last digit, "1" through "8", reports the strike price position relative to the underlying futures price where "1" is the closest to the money and “8" is the furthest in- or out-of-the-money. For example, CI3 indicates an in-the-money call option whose strike price is the third strike below the futures price.[13] In Table 4 we only report results for strikes traded on 75 or more days and in Figure 2 we only show results for strikes between CI4 (or PO4) and CO4 (or PI4).

As shown in Figure 2 and Table 4, implied volatilities in this market display a standard smile pattern - generally rising as strikes further from the underlying futures price are considered. The smile is considerably steeper at the shorter maturity.

In Figure 3 we report how implied volatility in this market tends to vary with the time-to-expiration. For this we calculate the average implied volatility each day on the four at-the-money options: CO1, CI1, PO1, and PI1 for each option expiry. These are then averaged for each option expiry: the nearby options (but at least two weeks to expiry), the next longest options, etc. As shown in Figure 3, implied volatility generally rises with the time to expiration.

IV. Volatility Trading.

In Table 5, we report the number of trades in our data set for each of our six volatility spreads. This figure is reported as a percentage of (1) all trades of 100 contracts are more, (2) all spread and combination trades (i.e., removing naked calls and puts), and (3) the six volatility trades. In panel B we report the same percentages based on trade volume. Confirming their importance, the six volatility trades account for about 24% of all trades of 100 contracts or larger and about 23.3% of the trading volume due to these large trades. They represent about 41.5% of all spread/combination trades.

In section II, we argued that based on gamma, vega, delta, and transaction costs, straddles should normally be the most attractive of the six volatility trades followed by strangles. Consistent with this, straddles are by far the most popular of our six volatility trades accounting for 73.1% of the trades in our volatility spread sample and 58.9% of the volume. Strangles are second at 20.8% of the trades and 28.0% of the volume. Butterflies, which have low gammas and vegas and relatively high transaction costs, are a distant third at

4.7% of the trades and 10.6% of the volume. Not surprisingly based on the discussion above, guts, iron butterflies, and condors are very rarely traded collectively accounting for less than 2% of trades and 3% of volume. Consequently, we restrict the remainder of our analysis to straddles, strangles, and butterflies.

In Table 6, we present statistics on aspects of the observed straddle, strangle, and butterfly trades: net (absolute) price, time-to-expiration, size, and implied volatility. We also present statistics on the estimated Black and BW model Greek values. For these calculations we remove from the sample: (1) those straddles and strangles with a simultaneous futures trade, (2) those expiring within two weeks, (3) all mid-curve options (because the underlying futures price was unavailable), and (4) a few with incomplete data. These restrictions, especially removing the midcurve options, reduces the sample about 26%.

Consistent with our discussion above, straddles have the highest net price and butterflies the lowest. Straddles also tend to have considerably longer expiries than strangles and butterflies. Median trade size (conditional on only observing trades of 100+ contracts) is 1000 contracts for both straddles and strangles and 2000 for butterflies but mean trade size is considerably higher for strangles than for straddles.

Calculating implied volatilities and the Black and BW model Greek values requires (in addition to the spread price and time-to-expiration from our data set): the interest rate and the underlying asset price. For the interest rate, we utilize the rate on the T-bill or note which expires closest to the options' expiration date.[14] Unfortunately, we do not know the exact price of the underlying Eurodollar futures at the time of the option trade since neither that information nor the time of the trade are recorded by Bear Brokerage's observer. Consequently, we approximate the futures price using an average of the open, settlement, high, and low prices that day. The implied volatilities reported in Table 6 are those which equate the actual price of the volatility spread with the Black price. Reliable implied volatility figures could not be calculated for the butterfly spreads because (due to their small vegas) their prices proved fairly insensitive to the chosen volatility. Because the signs of the

Greeks depend on whether the combination is long or short, the reported statistics are for absolute values.

As shown in Panel B of Table 6, with median absolute Black deltas of .109 and .104 respectively and means of .156 and .135, most straddles and strangles have low deltas but are not completely delta-neutral. Butterflies tend to have somewhat lower absolute deltas (mean = .074). On the other hand (and as expected from the analysis in section II), gamma and vega are much higher for straddles and strangles than for butterflies. Indeed, most straddle gammas and vegas are close to the maximum possible levels for that expiry. Since gamma and vega vary with time-to-expiration, it is more instructive to examine the figures for $n(d)'$ than gamma and vega directly. Recall from section II and Table 2 that for a given expiry, $n(d)'$ is maximized at .7979 if the straddle is constructed with a strike equal to $X = Fe^{.5\sigma^2 t}$. The median value of $n(d)'$ for straddles is only slightly less than this theoretical limit at .790 while the mean is .766. Consequently for most straddles, gamma and vega are close to their maximum values. To compare vega and gamma for straddles and strangles, it is again more instructive to compare values for $n(d)'$ than to compare gamma and vega directly since their expiries differ. With a median $n(d)'$ = .699 and mean =.629, $n(d)'$ (and therefore gamma and vega) tend to be about 10 to 15% smaller for strangles than for straddles with the same expiry. As expected from our analysis in section II, $n(d)'$, gamma, and vega tend to be much lower for butterflies than for straddles and strangles. Indeed, due to the low vegas, we cannot obtain reliable implied volatility values from the observed butterfly prices so these are not reported in panel A. As compared with straddles and strangles, butterflies are clearly quite weak volatility plays which probably explains why we observe relatively few.

In Panel C we present average and median absolute Greeks according to the Barone-Adesi-Whaley (BW) model. As shown there, delta, gamma, and vega are virtually identical to the Black model estimates and this is true for virtually all of the individual spreads. Because the BW prices are somewhat higher reflecting the possibility of early exercise, the thetas tend to be somewhat larger.

IV. Straddle Design.

We next explore the design of the two most popular volatility trades, straddles and strangles starting with straddles. First, we analyze how a straddle's design impacts such characteristics as its delta, gamma, and vega., then the resulting hypotheses are tested. For our analysis, we again primarily utilize Black's (1976) futures options model as presented in Table 2. However, for most of our empirical work, we check to see if the same relationships hold for the Barone-Adesi-Whaley (BW) American options model as in Tables 3 and 6.

IV.1. Straddle Design Issues.

Straddles are among the simplest of combinations since, in constructing a straddle for a given expiry, the straddle trader faces only one design choice: which strike price to use. Except for Natenberg (1994), who notes that delta is approximately zero if the straddle is at-the-money, we find no discussions of this question in the literature though most examples involve at-the-money straddles.

We hypothesize that straddle traders will tend to choose constructions which maximize the straddle's sensitivity to changes in actual and/or implied volatility (gamma and vega) and minimize sensitivity to the price of the underlying asset (delta). As we have already shown in section II, according to Black's model, if a straddle trader wishes to construct a straddle which is both delta neutral and highly sensitive to changes in actual or implied volatility, she should choose a strike slightly above the current futures price. More precisely as shown in section II, a straddle's delta is zero (since N(d)=0) and vega and gamma are maximized (since n(d) is maximized at .3989) when the strike price is equal to $F^* = F e^{.5\sigma^2 t}$ where F is the underlying futures price, σ is the instantaneous volatility and t is the time-to-expiration.[15] For short time-to-expiration options, the exponential term is small so F* is just slightly above the current futures price, F. In our straddle data set, the mean of F*-F is 7.3 basis points. For most straddles, the strike at which delta is equal to zero according to the BW model is virtually identical.

At strikes below F*, a bought (sold) straddle's delta is positive (negative) while it is negative (positive) for strikes above F*. As the strike is moved away from F* in either direction, gamma and vega are reduced (in absolute terms) since n(d) falls. The relationships between the chosen strike and delta, gamma, and vega according to the Black model are illustrated in Figure 4, where we graph a straddle's delta, and gamma/vega (or n(d)▯) as functions of the strike price for the case when σ = .16, t = .5 years, r=.065 and F=6.50. For ease of comparison, these are shown in relative rather than absolute terms. Each is expressed as a percentage of the derivative's maximum value so that gamma and vega, and theta vary from 0 to1.0 and delta from -1 to +1.

Unfortunately, since only a limited number of strikes are traded, straddle traders can rarely choose a strike exactly equal to F* so cannot make their straddle completely delta neutral (and cannot completely maximize gamma and vega). In the Eurodollar option market, options which expire in less than three months are currently traded in strike increments of 12.5 basis points for the five or so strikes closest to the underlying futures and in 25 basis point increments for the strikes further from the money. Options with expiries exceeding three months are all traded in increments of 25 basis points.[16] Suppose, as in our example above, σ = .16, t = .5 years, and r=.065 and suppose F=6.60, so F*=6.642. The closest available traded strikes are 6.50 and 6.75. If X=6.50, the Black delta of the straddle is +0.147. If X=6.75, delta= -0.109. According to the BW model, delta is +0.156 when X=6.50 and -0.130 when X=6.75. So even if the trader chooses the strike closest to F*, some delta risk remains. Since the exact strike at which delta is zero and gamma and vega are maximized, F*, is normally unavailable in reality, we focus attention on the delta minimizing strike among the strikes actually traded, which we label X*. It is easily shown that X* is the traded strike which is closest to F* in log or percentage terms. We label F* the "zero-delta" strike, and refer to X* as the "delta-minimizing" strike. It is also the gamma and vega maximizing strike among the traded strikes since it is the strike closest to F*. For some comparisons, we also make use of the closest-to-the-money strike, X_m. So while X* is the strike closest to F* in log terms, X_m is the strike which is closest to F in log terms. In the literature, it is often referred to as the "at-the-money" strike although it is rarely exactly at the money. For 71.4% of our straddle observations, $X^*=X_m$.

The presumption that straddles traders seek to minimize delta and maximize gamma and vega leads to our first hypothesis:

> **H1: Ceteris paribus, straddle traders will tend to choose the strike X* which is the available strike at which the Black delta is minimized and the Black gamma and vega are maximized.**

Other objectives may lead to different choices. For instance, a long straddle trader who thinks actual volatility will exceed implied volatility but also thinks a rise in the Eurodollar rate is more likely than a decline may want to choose a strike <u>below</u> F* in order to obtain a positive delta even though X* may be above F*.

More important, in deriving H1, we have assumed that implied volatility, σ, is the same at every strike price. However, from Table 4 and Figure 2, we know that this is usually not the case. As discussed in section II.4, above, if implied volatility differs across different strikes and if traders view these differences as real, rather than the result of calculating implied volatility with the wrong model, then a long straddle trader may wish to long strikes with relatively low implied volatilities and avoid those with relatively high implied volatilities. Conversely, a short straddle trader may prefer to short strikes with relatively high implied volatilities. Hence our second hypothesis is:

> **H2: Traders will tend to choose strikes with relatively low implied volatilities for long straddles and strikes with relatively high implied volatilities for short straddles.**

H2 assumes that the implied volatilities are correctly calculated. If the true implied volatilities are all the same and the implied volatilities calculated using the Black model only differ because of deficiencies in the Black model, such as the constant volatility assumption or the assumption that log returns are normally distributed , then there is no reason to expect strike choice to be related to the apparent implied volatility pattern.

IV.2. Straddles: Results

Results relevant to H1 are presented in Table 7. Clearly almost all straddles are constructed using close-to-the-money strikes. More than 75% are constructed using either the strike immediately above or below the current futures price. More than 95% were constructed using either the at-the-money strike or the strike on either side of the at-the-money strike. Clearly, straddle traders are choosing strikes at which delta is relatively low. However, they are not always choosing the delta-minimizing strike, X*. 54.3 % of the 1751 straddles were constructed using X*. However, in 71.0% they chose the strike closest to the current futures price, X_m. As noted above, in 71.4% of our observations, X* = X_m. If we restrict attention to the 435 observations when X*…X_m <u>and</u> the trader chose one or the other, in 364 or 83.7% of these cases, the trader chose the closest-to-the-money strike, X_m, rather than X*. In only 16.3% was the delta minimizing strike, X*, chosen. Hence, although the evidence indicates that straddle traders choose strikes with fairly low Black deltas, hypothesis H1 (that they minimize the Black delta) is rejected.

What are the consequences of choosing X_m instead of X*? In the 364 cases in which the trader chose X_m rather than X*, the average absolute delta was .115. If instead, they had chosen X*, it would have been only .052 according to the Black model. In terms of the BW model, the estimated delta is .118 while it would have been only .054 if the X* strike had been chosen. While this delta difference is statistically significant at the .0001 level, whether it is economically important is in the eye of the beholder. On the one hand, in those cases when traders chose X_m instead of X^*, they could have reduced their delta risk by more than half by choosing X* instead. On the other hand, at X_m the straddle's delta is fairly low anyway. It makes less difference in terms of gamma and vega whether they choose X* or X_m. Specifically, they would only raise vega and gamma about 0.9% by switching from X_m to X*.

Clearly, the hypothesis H1, that straddle traders tend to choose the delta-minimizing and gamma-vega-maximizing strike is rejected. Moreover it is clear that most, specifically 71.05%, straddle traders tend to choose the strike which is closest to the current futures price,

X_m. At this strike, delta is low but not always at its minimum level according to the Black and BW models.

While it is clear that <u>most</u> traders tend to choose the closest-to-the-money strike, this is not true for all. In 436 or 24.9%% of our observed straddles, the chosen strike is neither X* nor X_m. In 83.9% of these, the straddle trader chose one of the next closest strikes, e.g., 5.75 or 6.25 if $X^*=X_m=6.00$ and the time-to-expiration exceeds three months. In these, the average absolute Black delta was .299 versus .097 if they had chosen X* so these traders were taking substantial delta risk with their strike choice. In only 70 of the 1751 cases or about 4% of the straddles, was a strike more than one strike from X* and X_m chosen. In these the average absolute delta was .503 - roughly the same delta which would have been obtained if the trader had purchased an at-the-money naked call or put instead of a straddle.

In summary, most (71%), but not all, straddles are constructed using the closest-to-the-money strike. In a majority of cases, this is also the strike at which delta is minimized and gamma and vega are maximized according to the Black and BW models. However, when faced with a choice between the closest-to-the-money strike and the delta-minimizing strike, most straddle traders choose the strike which is closest-to-the-money even though by doing so they accept some delta risk (and slightly lower gammas and vegas) according to the Black model. In the minority of cases when neither the closest-to-the-money nor the delta-minimizing strike is chosen, the delta risk is substantial.

To test H2 we focus on the 24.9% of our straddle observations cases when the trader chose a strike other than X* or X_m. According to H2, if volatility traders are shorting volatility, i.e., betting that actual volatility will be less than implied or that implied volatility will fall, they should prefer to short a strike with high implied volatility. Conversely, if longing volatility, they would prefer a strike with low implied volatility. Let ISD be the implied standard deviation of the straddles at the chosen strike ,X, and ISD* and ISD_m be the implied standard deviations at X* and X_m. Consider the cases when X...X* and X...X_m. According to H2 for a short straddle, $ISD>ISD^*$ and $ISD>ISD_m$ while for a long straddle we expect $ISD<ISD^*$ and $ISD<ISD_m$. Unfortunately, we cannot observe ISD* and ISD_m at the

exact time of the trade since the time is not recorded. Consequently we compare implied volatilities calculated from the previous day's settlement prices viewing these as providing the signal which is executed on day t. ISD, ISD*, and ISD_m are available for 388 of our 436 observations.[17]

Contrary to H2, we observe no significant different between ISD and ISD* or between ISD and ISD_m. For the 206 short straddles, the mean ISD is .1605 while ISD* is .1593 and ISD_m is .1589. The differences are insignificant at any reasonable significance level. Results for the 182 long straddles are similar; the means are .1607, .1599, and .1598 for ISD, ISD*, and ISD_m respectively. The results are basically unchanged if the ISDs are calculated using day t prices instead of day t-1 or if we calculate ISD using the actual price of the straddle. Hence, H2 is not confirmed . Again, however we would caution that the power of this test is weak since we cannot observe the ISDs at the exact time of the trade.

IV.3. Straddles with Futures.

To this point, we have excluded from our analysis those straddles which were accompanied by a simultaneous futures trade which constitute about 8.5% of Eurodollar straddle trades.[18] We hypothesize that the reason straddle traders combine the straddles with a futures position is in order to achieve delta-neutrality. Adding futures to a straddle changes the position's delta but does not affect its gamma, vega, or theta. Consequently, a straddle trader can choose the strike with the desired ISD or gamma-vega-theta characteristics and then use futures to lower the straddle's delta. Hence, we hypothesize:

H3: If futures are traded simultaneously with a straddle, the straddle position's absolute delta including the futures will be lower than it would be without the futures. In other words, futures will be bought (sold) when the straddle's delta (ignoring the futures) is negative (positive).

Taking this idea further we also hypothesize,

H4: Futures will be bought (sold) in quantities which reduce the position's delta approximately to zero.

This second hypothesis is a stronger version of the first considering the size of the futures position as well as its sign.

If the purpose of the futures trade is to achieve delta neutrality, we would expect this strategy to be employed when the straddle's absolute delta without the futures is relatively high, which occurs when the chosen strike is far from X*. Consequently, our third hypothesis is:

H5: Those straddles accompanied by a simultaneous futures trade will tend to be at strikes further from F* and to have higher absolute deltas than the straddles traded without futures.

IV.4. Straddles with Futures: Results.

To test H3, we compare deltas with and without the futures for those straddles which were accompanied by a futures trade. Removing those observations involving midcurve options, options maturing in less than two weeks, and trades where the size of the futures trade is unrecorded, our sample consists of 153 such straddles. Confirming H3, in 142 or 92.8% of these 153 observations, the straddle position's absolute delta is reduced by incorporating the futures trade. Moreover, eight of the eleven contrary trades in which the futures actually increased the position's absolute delta were placed by the same clearing firm. It appears that this firm, or one of its customers, was following a unique trading strategy whose objective is unclear.

Turning to the question, H4, of whether adding the futures reduces the straddle position's delta to zero, we focus on the 142 observations in which delta is reduced. Calculated without the futures, the mean absolute delta of the straddles alone is .264. When the combined delta is calculated including the futures, the mean absolute Black delta is only .038. The median absolute delta without the futures is .202 whereas with the futures it is only .023. It seems clear that the purpose of combining a futures trade with a straddle is to reduce the position's Black delta to close to zero and that this strategy is successful.

The goal of reducing the Black delta to near zero levels is also apparent in how the trades are constructed. All but a couple of our straddle trades are in increments of 50 options, e.g., 250 or 300 options, and most are in increments of 100. However 83.1% of the futures trades are in smaller increments. For instance, in one case 86 futures contracts are traded with 200 straddles. Calculated without the futures, the straddle's Black delta is .427; with the futures it is -.003. In another case, 765 futures are traded with 1700 straddles. Without the futures, the straddle's delta is .4428 but with only -.008.

Finally, we hypothesized (H5) that straddles traders choose to combine futures with their straddles when the chosen strike price is far in- or out-of-the-money, or more precisely when it is far from F*, so that the absolute delta of the straddle alone is high. For the 904 straddles which were not accompanied by a futures trade, the average absolute difference between the chosen strike price X and the zero-delta strike price F* is13.8 basis points. For the straddles accompanied by a futures trade, the mean difference between X and F* is 30.9 basis points or over twice as far from F* on average. The difference is significant at the .0001 level so H5 is also confirmed. For the straddles unaccompanied by a futures trade, the mean absolute delta is .156. For the straddles accompanied by a futures trade, the mean absolute delta calculated without the futures is .264. Again the difference is significant and H5 is confirmed.

In summary, we find that straddle traders tend to combine their straddles with a futures position when the chosen strike is far in- or out-of-the-money so that the straddle alone is far from delta neutral, i.e., when they are exposed to substantial risk from a change in the price of the underlying asset. In these cases, traders tend to long or short futures in quantities which reduce the Black delta of their combined position approximately to zero.

V. Strangles

V.1. Strangle Design Issues

In a long (short) strangle, the trader buys (sells) a call at one strike price and buys (sells) a put at a lower strike price so the trader faces two strike price choices: one for the call and another for the put. These may be viewed as a choice of (1) the differential or gap between the put and call prices (so that a straddle becomes a special case of a strangle with zero gap) and (2) the relation of the two strikes to the futures price.

Holding the call-put strike differential constant, consider first the question of the distribution of the strikes around the underlying asset price. Note from Table 2 that a strangle's Black delta is zero iff $N(d_c)+N(d_p)=1$. Since $N(-x)=1-N(x)$, this occurs when iff $d_c = -d_p$ where d_c represents d defined in terms of the call strike X_c and d_p = d defined in terms of the put strike, X_p. If the volatility is the same for both the call and put, the $d_c = -d_p$ condition is met when $\ln(F/X_c) + \ln(F/X_p) = -\sigma^2 t$ yielding the result that the strangle delta is zero iff $(X_c X_p)^{.5} = F^*$ where $F^* = F e^{.5\sigma^2 t}$ as before. In other words, a strangle's delta is zero iff the geometric mean of the two strikes, which we will designate as $\bar{X}$, equals F*. As is also apparent from the expressions in Table 2, for a given gap between the two strikes, gamma, and vega are maximized when $\bar{X} = F^*$. Again, since the traded strikes are in increments of 12.5 or (more normally) 25 basis points, a strike pair whose geometric mean is exactly equal to F* is not normally available. It is easliy shown that for a fixed differential, the Black delta is minimized by choosing the pair whose geometric mean is closest to F*. Consequently, the presumption that strangle traders wish to minimize their position's exposure to changes in the value of the underlying asset while maximizing exposure to volatility implies:

H6: For a given strike price differential, strangle traders will tend to choose the strike price pair at which the Black delta is minimized which is the pair whose geometric mean is closest to F*.

Note that if the difference between the two strikes is the minimum possible, hypothesis H6 implies that they will choose the pair of strikes which bracket F*.

Given our straddle results above, in which traders tended to choose X_m instead of X* for the strike price, an obvious alternative to H6 is that the strikes will be constructed so that $\bar{X}$ is approximately equal to F, not F*: Therefore we also examine the alternative:

H6b: For a given strike price differential, strangle traders will tend to choose the strike price pair whose geometric mean is closest to F.

Next attention is turned to the choice of the differential or gap between the two strike prices X_c and X_p. Note that because a straddle can be viewed as a strangle with a zero differential, this analysis applies to the straddle/strangle choice as well. Consider first the impact on the price and expected payout. Since the payoff on a strangle is zero if the final asset price is between the two strikes, increasing the gap between the two strikes in a strangle while holding the geometric mean constant,[19] clearly lowers the expected payout. Since the price of the strangle is the discounted value of the expected payout, the current price is reduced an equal percentage. This is illustrated in Figure 5 where we graph the net price and expected payoff of a strangle, according to the Black model, for different (assumed continuous) strike price differentials for the case when F*=6.50, r=.065, σ =.16, t=.5 and holding $\bar{X}$ = F*. While the Black price of a long straddle with these characteristics is 57.0 basis points, a strangle with a 25 basis point differential costs 45.8 bp, and with a 50 basis point differential costs only 36.1bp. As in Figure 4, to aid comparison, the price is expressed as a percentage of its maximum value which occurs when the gap = 0, a straddle. Since, in percentage terms, the impact on price and the expected payout is the same, this should not be an important consideration to most strangle traders.

Of more interest for our purposes is the impact of a change in the strike price differential on the Greeks. If the geometric mean is unchanged at F*, increasing the call-put strike differential leaves delta unchanged but tends to reduce gamma and vega. As shown in Table 2, for constant volatility, and expiry, a strangle's gamma and vega are proportional to $[n(d_{1c})+n(d_{1p})]$. Consequently, if the call and put prices bracket F*, then increasing the call-put differential while holding $\bar{X}$ constant reduces both $n(d_{1c})$ and $n(d_{1p})$. This is also illustrated in Figure 5 for the aforementioned case where F=6.50, r=.065, σ =.16, t=.5 and $\bar{X}$ = F*=6.54. While gamma is 1.05 for a long straddle with these characteristics, it falls to 1.03 for a strangle with a 25 basis point differential, to .99 for a strangle with a 50 basis point differential , to .92 for a 75 bp differential, and to .84 for a 100 bp differential. The impact on vega is proportional.

This analysis implies that if strangle traders wish to maximize their strangle's sensitivity to actual volatility (gamma) and implied volatility (vega), they should minimize the strike price gap, which at the extreme means choosing a straddle instead. As documented in Table 5, this is indeed normally the case, i.e., straddles are more than three times as common as strangles. However, this leaves open the question of why strangles are ever chosen and when they are why the gap is not always the smallest possible. One factor to note is that as illustrated in Figure 5, due to the shape of the normal density function, gamma and vega tend not to change much until the strike gap becomes fairly large. Consequently, a strangle/straddle trader may see little gamma/vega reason to prefer one small strike gap, such as zero bp, to another, such as 25 bp.

The implied volatility smile documented in Table 4 and Figure 2 suggest that implied volatility differences could be a determinant of the strangle strike gap choice. As illustrated in Figure 2, implied volatilities are normally lowest for the near-the-money strikes and higher on strikes considerably in- or out-of-the-money. If these implied volatilities are correct, that is if the implied volatility differences are not purely due to using an incorrect model for the calculations, then a trader wishing to speculate that actual volatility will be less than implied volatility should want to construct his straddle/strangle using strikes toward the top of the smile. If he constructs a straddle (without futures) using one of these strikes, the delta risk will be substantial. However, a strangle constructed using these strikes can be made delta neutral if $\bar{X}$=F*. Of course this situation is reversed for strangle/straddle traders who want to long volatility; they should normally prefer either a straddle or a strangle with a small gap at strikes at the bottom of the smile near F*. This yields our next hypothesis:

H7: Given a u-shaped volatility smile, traders will tend to construct strangles which are short volatility using large strike price gaps and long volatility strangles using small price gaps.

For liquidity reasons, we also expect the strike price differential to depend on the time to expiration. At short times to expiration, far in- and out-of-the-money options are thinly traded and likely to have large spreads.[20] A strangle with a 100 bp differential which is easy to trade when the time to expiration is six months may be very hard to trade when the expiry is within a month. Accordingly, we hypothesize:

H8: The strike price gap on a strangle will tend to be directly related to the time to expiration.

V.2. Strangle Results.

The results for hypotheses H6 and H6b match our straddle results. Most strangle traders choose strikes whose geometric mean, $\bar{X}$, is close to the current futures price whether or not this is the Black delta minimizing pair. In other words, H6 is rejected in favor of H6b. For a given strike price differential, in 57.0% of our observations, strangle traders choose the strike pair at which the Black delta is minimized. However, in 67.2% they choose the pair whose geometric mean is closest to F. The average difference between the geometric mean and F* is -9.8 basis points while the average difference between the geometric mean and F is considerably lower (though still significantly different from zero) at -2.3 basis points. In 63.2% of the observations, the geometric mean strike is closer to F than it is to F*, a percentage which is significantly greater than 50% at the .0001 level.

According to the Black model, a straddle's Black delta is minimized by choosing a strike as close to F* as possible while a strangle's delta is minimized by choosing strikes whose geometric mean is as close to F* as possible. Our results for both straddles and strangles indicate that instead straddle/strangle traders seek strikes which are close to the underlying futures, F, a strategy which yields a low delta though not always the smallest. In other words, traders seem to be following a simpler rule of thumb which works well, if not perfectly.

According to our analysis above, to minimize the Black delta, strangle traders should compare the strikes' *geometric* mean with F*. If traders are applying a simpler rule of thumb to the latter half of this equation, one wonders whether they are applying a similar rule of thumb to the other half as well. Consequently, we ask if they tend to choose the strike pair whose *arithmetic* mean, rather than the geometric mean, is as close as possible to F. Evidence indicates that a majority are. In 54.3% of our observations the arithmetic mean is closer to F than the geometric mean - a proportion that is significantly greater than 50% at the .05 level. In summary, most straddle traders seek strikes which are approximately equal distances from the underlying asset's price, a strategy which results in a low absolute Black (and BW) delta - but not always the minimum possible delta.

Turning to the strike price differential question, characteristics of strangles stratified by the strike price differential are reported in Table 8 where we also repeat the straddle figures from Table 6. We divide the 530 strangles into the following strike gap buckets: 25 basis points or less (mostly 25), 26 to 50 bp (mostly 50), 51 to 100 bp (mostly 75 or 100), and over 100 bp. Conclusions regarding hypotheses H7 and H8 depend on whether or not straddles are included in the sample. In H8 we hypothesized that because of the supposed illiquidity of far-from-the-money strikes at short expiries, the strike price gap and the strangle expiry would be positively related. Excluding straddles, this hypothesis is confirmed. As reflected in Table 8, time to expiry and the strike price gap are positively and significantly (.0001 level) correlated. Straddles, which of course may be viewed as strangles with a zero price gap, fail to follow this pattern. Their average time to expiration is 7.94 months versus 5.04 months for strangles in general. Since, H8 was based on the supposed illiquidity of far-from-the-money strikes, it is not surprising that it would fail to hold for straddles but the fact that straddles actually tend to have longer expiries is. A possible explanation is explored in section VI below.

For H7, the reverse is true: the evidence tends to support H7 if we include straddles but not if we examine strangles only. In H7 we hypothesized that because of the U shaped volatility smile, traders would tend to choose large strike gaps for short volatility positions and small gaps for long volatility positions. Consistent with H7, the percentage of short positions is 53.9% for straddles and 63.0% for strangles - figures which are significantly different at the .0001 level. However, the pattern within the strangle category is not consistent with H7. As shown in table 8, the percentage of short positions is fairly constant for gaps of 25 to 100 basis points (actually declining slightly) and then falls sharply for the largest gaps of over 100 basis points. Since the latter should represent gaps far up both sides of the smile, these are the strangles which we would have expected to be most heavily short but long positions actually predominate slightly.

Differences in time-to-expiration make univariate tests of H7 suspect. As shown in Table 4, the smile is more pronounced at shorter maturities than at long so a gap of say 25 basis points is fairly far up the sides of the short-term smile in Figure 2 but near the bottom of the long-run smile in Panel b of Table 4. Since time to expiration tends to be longer for the larger gaps in Table 8, this could conceivably explain why H7 is not confirmed in Table 8. To test H7 controlling for time-to-expiration, we regress (excluding straddles) the chosen strangle strike price differential, GAP, on (1) the time-to-expiration, TIME, and (2) a buy/sell dummy, B/S which = 0 if the strangle is bought and =1 if sold yielding:

$$\underset{}{GAP} = \underset{(11.16)}{45.7} + \underset{(8.93)}{.611}\ TIME - \underset{(13.75)}{14.7}\ B/S$$

where t-statistics are shown in parentheses. Hence, the chosen strike price differential is significantly related to both strangle's expiry and to whether the trader buys or sells the straddle but the latter relationship is opposite to our hypothesis 7. For the same expiry, the gap tends to be about 14.7 basis points smaller if the straddle is sold. These results confirm H8 and reject H7.

Hypothesis H7 was based on the presumption that short volatility traders would seek strikes with relatively high implied volatilities while long volatility traders would seek strikes with relatively low implied volatilities. To explore this question more directly, we compared the average implied volatilities for the strangles with a spread of 50 basis points with (1) the implied volatility on a straddle with the same midpoint and (2) a strangle with a 100 basis point strike differential. For example if the strangle's strikes were 6.00 and 6.50, we compared the average implied volatility of these two strikes with the implied volatility at a strike of 6.25 and at the 5.75-6.75 strike pair. This was done separately for short and long positions. No significant differences were observed. In summary we find no evidence that the strangle trader's strike choice is related to implied volatility differences - the smile.

As we have seen above, for a given expiry and mean strike, the price and expected payoff on a strangle are inverse functions of the strike price differential. Consequently, an alternative hypothesis consistent with the observation that strangle gaps tend to be smaller on long positions would be that traders who are buying strangles (long positions) seek to minimize the price by making the strike gap large while those who are selling strangles (short) seek to maximize the price by making the gap small. Of course, for the same expiry, the expected payoff at expiration is proportional to the price so the price advantage is offset by an expected payout disadvantage. Also while the strangle results are consistent with the price minimization argument, the straddle results are not.

Aside from hypotheses 7 and 8, there are other interesting patterns in Table 8. First, for all gaps, the deltas are fairly low reflecting the fact that whatever the size of the gap, strangle traders tend to center the strangle at F. Second, as it must, $n(d)_c$ declines as the gap is increased falling from .766 for gaps of 25 basis points or less to .456 for gaps of more than 100 bp. For a given expiry, this means lower gammas, vegas, and thetas as the gap is increased. However, since vega is a positive function of time to expiration while gamma and theta are inverse functions, they do not decline monotonically in Table 8.

VI. Straddles versus Strangles.

VI.1. Determinants of the straddle/strangle choice.

Finally, we look more closely at the straddle/strangle choice question. Since a straddle may be viewed as a strangle with a zero strike price differential, the discussion in section V.1. of the strangle strike gap also applies to the straddle-strangle choice issue. Based on that analysis, we would expect straddles to be more popular in general since delta neutral straddles have larger gammas and vegas than delta neutral strangles with the same expiry. Similarly, applying hypothesis H7 to the straddle/strangle choice decision and assuming the usual U-shaped smile implies that strangles should be relatively more attractive for short positions and straddles for long..

The fact that strikes are only traded in increments of 12.5 or 25 basis points introduces another element into this choice. If F* is close to one of the traded strikes, then a straddle at that strike will have a lower absolute delta than a strangle using that as one of the strikes. However, if F* is approximately midway between two strikes, a strangle based on those two strikes will have a lower delta than a straddle based on either strike alone. For instance, suppose $\sigma = .16$, t =.500 years (6 months) , r=.065, and F=6.60 so F*= 6.64. If X=6.50, the delta of the straddle is 0.147. If X=6.75, the straddle's delta is -0.109. However, since the geometric mean of the two strikes is 6.62 which is close to F*= 6.64, the delta of the strangle using these two strikes would be only 0.019. Presuming that approximate delta neutrality is important to volatility traders, we expect them to choose the combination, straddle or strangle, with the lowest absolute delta yielding the hypothesis:

H9: Volatility traders will tend to choose a straddle when F* is close to a traded strike and a strangle when F* is approximately midway between two traded strikes.

Given our previous results, we also test an alternative based on F instead of F*:

H9b: Volatility traders will tend to choose a straddle when F is close to a traded strike and a strangle when F is approximately midway between two traded strikes.

To test H9 (H9b) we form a sample of all 124 (111) strangles with a strike price differential of 25 basis points where F (F*)is between the two strikes. Those cases when the strikes are quoted in 12.5 basis point increments are converted to 25 basis point basis by doubling the spreads between F (or F*) and each strike.. We then divide the 25 basis point differential between X_p and X_c into five 5 basis point regions: (X_p, X_p+.05), (X_p+.05, X_p+.10), (X_p+.10, X_p+.15),(X_p+.15, X_p+.20), (X_p+.20, X_c). According to H9 (H9b), we should observe more strangles when F* (F) is falls in the middle quintile, (X_p+.10, X_p+.15), and few when it falls in the first and fifth quintiles. If the null that the straddle-strangle choice is unrelated to whether F or F* is close to or between traded strikes is correct, then the strangles should be roughly equally distributed over all five quintiles. Results are reported in Figure 6. The distribution of F* relative to the strikes conforms to H9 in that we observe relatively few strangles with F* in the first and fifth quintiles and the null that F* is randomly distributed across the quintiles is rejected at the .01 level . However, contrary to H9 there are more observations in the fourth quintile than in the third and more in the fifth than in the second.

The data are more consistent with H9b. There are very few observations in which F falls in the first and fifth quintiles and the distribution is reasonably symmetric. The null that the distribution is random is rejected at the .01 level.

Turning next to straddles, we form samples of at-the-money straddles defined as straddles where F (or F*) is within 12.5 basis points of the chosen strike, X, and form quintiles: (X-.125, X-.075), (X-.075, X-.025), (X-.025, X+.025), (X+.025, X+.075), (X+.075, X+.125). H9 and H9b imply that we should observe more straddles when F* or F respectively is close to X, i.e. the middle quintile, and few when F* or F is far from X, i.e., in the first and third quintiles. Results are shown in Figure 7. In both cases, the null that the straddles are randomly distributed is rejected at the .01 level but the results are clearly more consistent with H9b - that is traders are more likely to construct a straddle than a strangle when the underlying futures is close to a strike.

VI.2. Probit estimations of the straddle/strangle choice.

Finally, we test hypotheses regarding determinants of the straddle/strangle choice using probit estimations. Our choice variable is coded as 1 for strangles and 0 for straddles so a positive coefficient implies that an increase in the variable means that a strangle design is more likely to be chosen. According to hypothesis H9 (H9b) a straddle design is more likely to be chosen when F* (F) is close to a traded strike so to test these hypotheses we include the variables Z* = |F*- closest strike| and Z=|F-closest strike|. Hypotheses H9 and H9b imply positive coefficients for Z* and Z respectively. Since, hypotheses H9 and H9b apply to cases when the underlying futures is close to or between two strikes, to test these hypotheses, we restrict our straddle/strangle sample to (1) all strangles where the gap between the two strikes is 25 (or 12.5 adjusted to 25) basis points and either F or F* is between the two strikes, and (2) all straddles at <u>either</u> of the <u>two</u> (to avoid a sample selection bias) strikes closest to F or F*.

Due to the normal shape of the implied volatility smile, hypothesis H7 implies that strangles are more likely to be chosen if the trader is taking a short position. In other words, given that traders prefer straddles and strangles centered at the money, if taking a long position, they should prefer to long the at-the-money strike due to its low implied volatility so will choose a straddle. If taking a short position, they should prefer to short the away-from-the-money strikes. To do this and keep the position delta neutral, they would need to use a strangle. To test this we include a long/short variable, LS, which is equal to 0 for a long position and 1 for a short. H7 implies a positive coefficient. Since the sample is restricted to close-to-the-money strikes, we don't expect hypothesis H8 to be relevant here but given the time-to-expiration differences observed in Table 8, we include time-to-expiration, TTE (measured in years) as a control variable.

Results are presented in the column labeled Model 1 in Table 9. Hypothesis H9b is confirmed at the .001 level while H9 is not. In other words, traders tend to choose straddles when the underlying futures is close to a traded strike and a strangle when it is not. This is consistent with our hypothesis that traders seek designs which are close to (but not exactly) Black delta neutral.

Because delta is more sensitive to the difference between the strike price and the underlying asset price at shorter maturities (in other words gamma is larger at shorter maturities) we would expect Z (and/or Z*) to be more important at shorter expirations. For example, suppose as in our earlier examples, σ=.16 and r=.065. Suppose further the underlying LIBOR rate is such that F*=6.625 or halfway between the strikes of 6.50 and 6.75. At a three month expiry, the absolute deltas of straddles constructed using either strike are about .18 while a strangle based on the two strikes is roughly delta neutral. If the time-to-expiration is one year, the absolute deltas of the straddles using either strike are roughly half as large (.09) so whether the trader uses a straddle or a strangle is not as important. Accordingly we would expect the variables Z and/or Z* to be more important at shorter expiries. This could also explain why we tend to observe longer expiries on straddles - that traders normally prefer straddles but switch to strangles at the shorter maturities if the underlying asset price is roughly halfway between two strikes.

To test this hypothesis, we add the interaction variable TTE*Z to the probit. Given our hypothesis that Z matters more at shorter expiries, a negative coefficient is expected. Results are shown in the column labeled Model 2 in Table 9. There is weak evidence to support our hypothesis in that the interaction variable's coefficient is negative and significant at the .10 level.

Supporting H7, the coefficient of the L/S variable is positive and significant at the .05 level implying that volatility traders tend to choose strangles for short positions and straddles for long positions. Since this is the first evidence we have found indicating that volatility traders may prefer to long options with low implied standard deviations and short those with high ISD's we decided to explore it further. If traders view implied volatility differences as genuine, i.e., not due to calculation errors, and seek to exploit them, we would expect any tendency to use straddles for long positions and strangles for short positions to be stronger when the smile is steeply sloped. To test this, we measure the slope of the smile as the ratio of implied volatilities at away-from-the-money strikes to those at at-the-money strikes. Specifically, we calculate the average implied volatility, V_1, that day at strikes within 20 bp of the underlying futures, and the average implied volatility of all traded off-the-money strikes, V_2, and then calculate the smile slope, V_2/V_1.[21] We then define

SMILE = V_2/V_1 if the straddle/strangle position is long and

= $-(V_2/V_1)$ if the straddle/strangle position is short.

The hypothesis that the tendency to use straddles for long positions and strangles for short will be stronger when the slope of the smile is steep implies a negative coefficient.

Results are reported in the final column of Table 9. As shown there the SMILE variable is insignificant and has the wrong sign so there is no evidence that the tendency to use straddles for long positions and strangles for short is stronger when the slope of the smile is steep. Overall therefore we conclude that there is little evidence that implied volatility differences influence volatility trade design.

VII. Summary and Conclusions

Despite the fact that they are discussed in every derivatives text, are extensively covered in the practitioner literature, and are actively traded, volatility trades such as straddles, strangles, and butterflies have received no attention in the finance research literature. Using data from the Eurodollar options market we have attempted to fill this gap.

Our study had two main objectives. The first was to compare the properties of the various volatility trades and document their popularity. Based primarily on likely transaction costs and their "Greeks," we expected straddles to be most popular volatility trade followed by strangles. This hypothesis was confirmed with straddles accounting for 73% of volatility trades and strangles 21%. Butterflies were a distant third at roughly 5% while guts, iron butterflies, and condors collectively accounted for only about 1.5%.

Our second objective was to examine the design on the two more popular strategies: straddles and strangles. Here we find that achieving approximate delta neutrality is important to most traders of straddles and strangles. In constructing straddles, volatility traders tend to either choose the strike closest to the underlying futures price, F, which results in a low delta according to the Black and Barone-Adesi-Whaley (BW) models or to combine the straddle with futures in a ratio which achieves delta neutrality. Likewise, most strangle traders choose a configuration in which the mean of the two strikes is close to F, a strategy which achieves approximate delta neutrality. Finally, in choosing between a straddle and a strangle, we find that volatility traders tend to choose the strategy with the lower delta.

However, our results indicate that most traders seek only approximate delta neutrality. Faced with a choice between the strike or strikes closest to the futures price, F, and those closest to the zero delta price, F*, traders normally choose the strike (strikes) closest to F even though this strategy results in a slightly larger absolute Black or BW delta. We observe this behavior in both the way straddles and strangles are normally constructed and in the straddle-strangle choice.

In general, the straddle/strangle design which minimizes delta is also that which maximizes gamma and vega so traders are not faced with a tradeoff between these objectives.

We find little evidence that the slope of the smile influences straddle/strangle design, i.e., little evidence that traders tend to short options with high implied volatilities and long those with low implied volatilities. There is a tendency to use strangles for short positions and straddles for long positions which is what one would expect given the usual U-shaped smile in this market but this tendency appears unrelated to the slope of the smile. Moreover, there is no evidence that traders design straddles and strangles to exploit implied volatility differences. This finding that traders apparently do not view implied volatility differences as exploitable is consistent with the view that the implied volatility differences are an artifact of calculation using an incorrect or incomplete model.

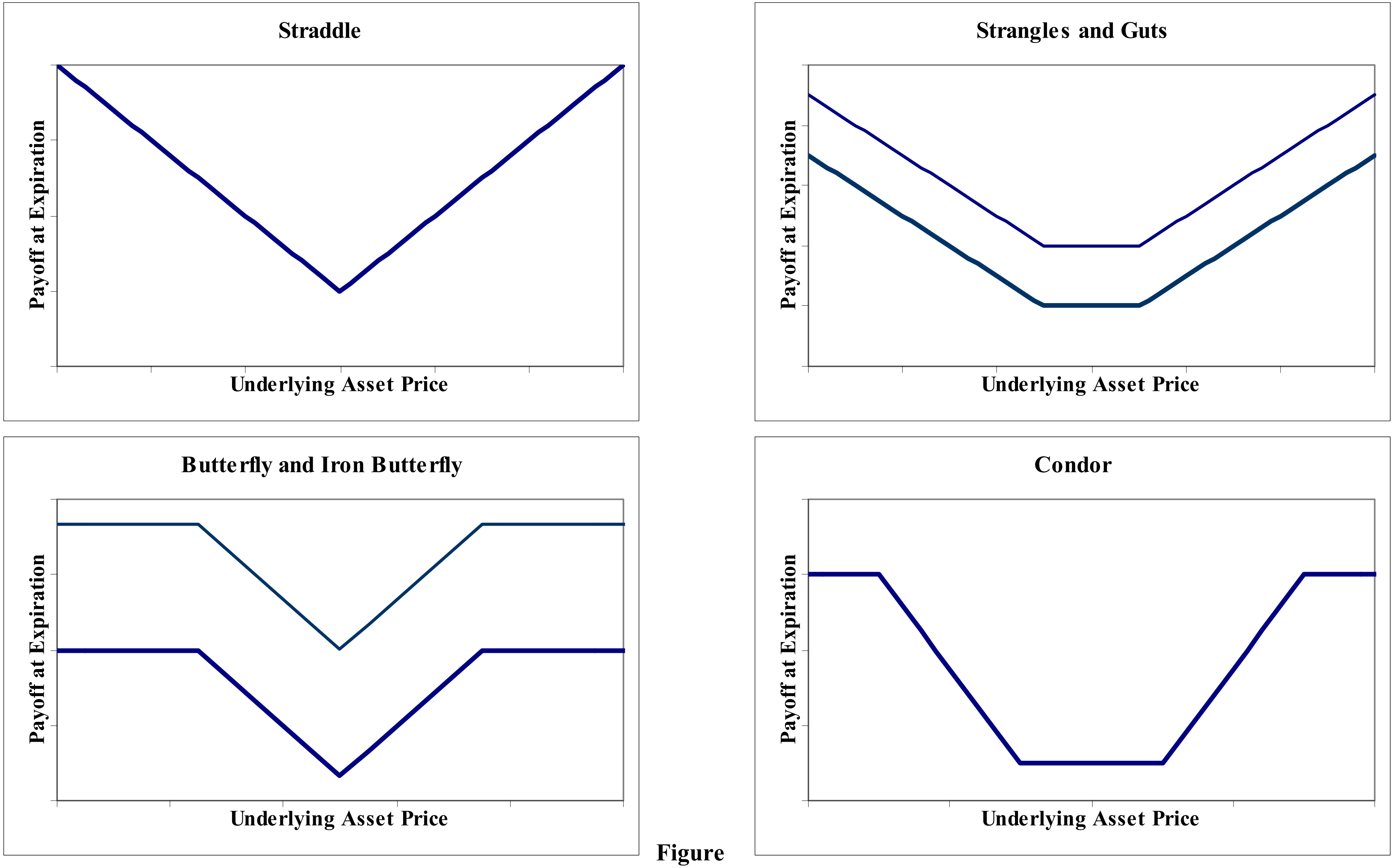

Figure 1 - Payoffs at Expiration on Volatility Spreads and Combinations. The payoffs at expiration to long volatility positions in six symmetric volatility spreads and combinations are shown as a function of the underlying asset's price.

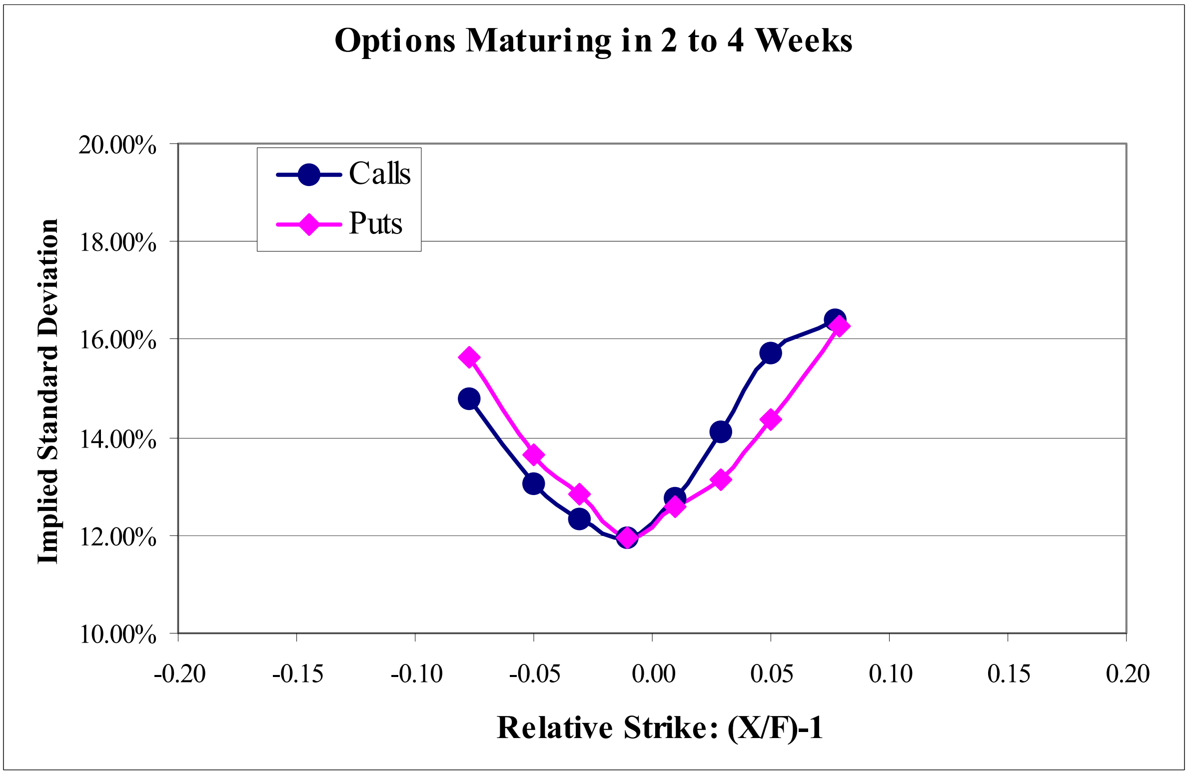

Figure 2 - The Implied Volatility Smile. Mean implied standard deviations at various strike prices are reported based daily data for the periods 5/10/94-5/18/95 and 4/18/99-7/31/00. The implied volatilities are those calculated by the CME from option and futures settlement prices. Strike prices are expressed in relative terms as (X/F)-1 where X is the strike price (in basis points) and F is the underlying futures price (in basis points).

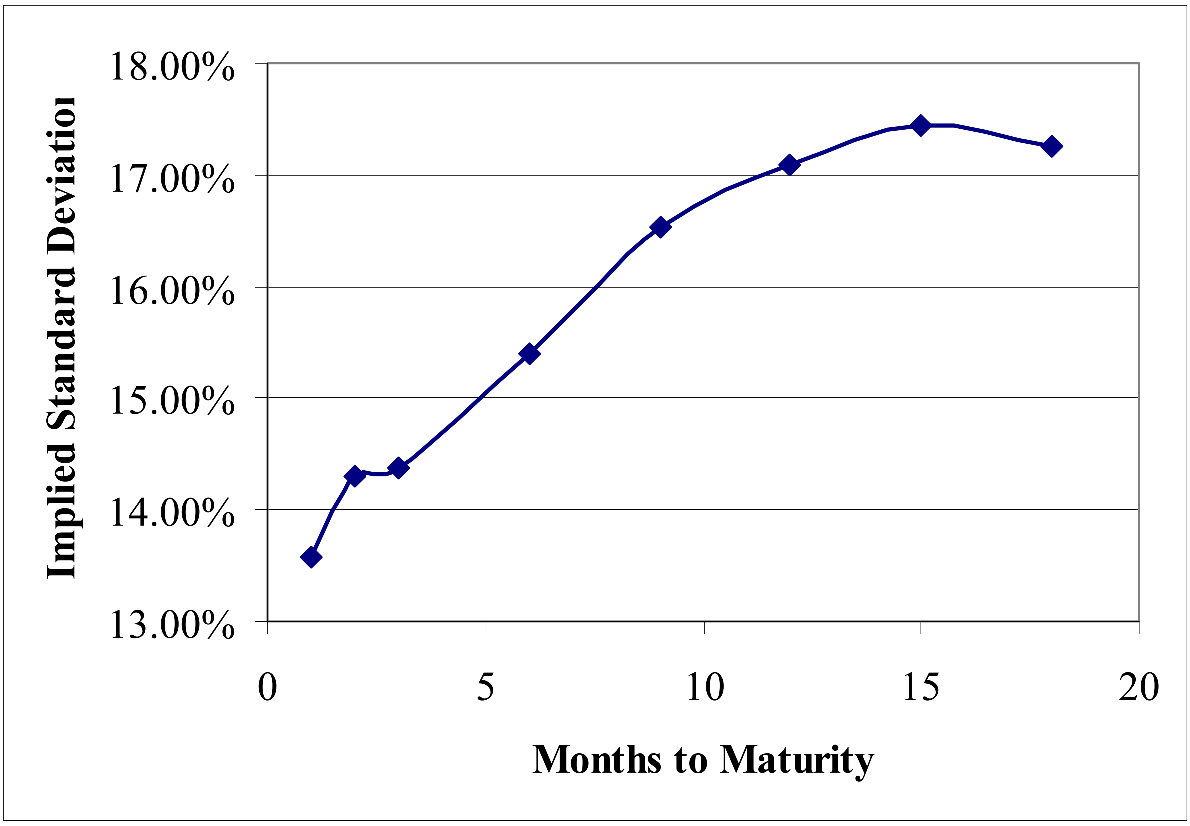

Figure 3 - The Implied Volatility Term Structure. Mean implied standard deviations at various times to expiration are reported based daily data for the periods 5/10/94-5/18/95 and 4/18/99-7/31/00. The implied volatilities are those calculated by the CME from option and futures settlement prices.

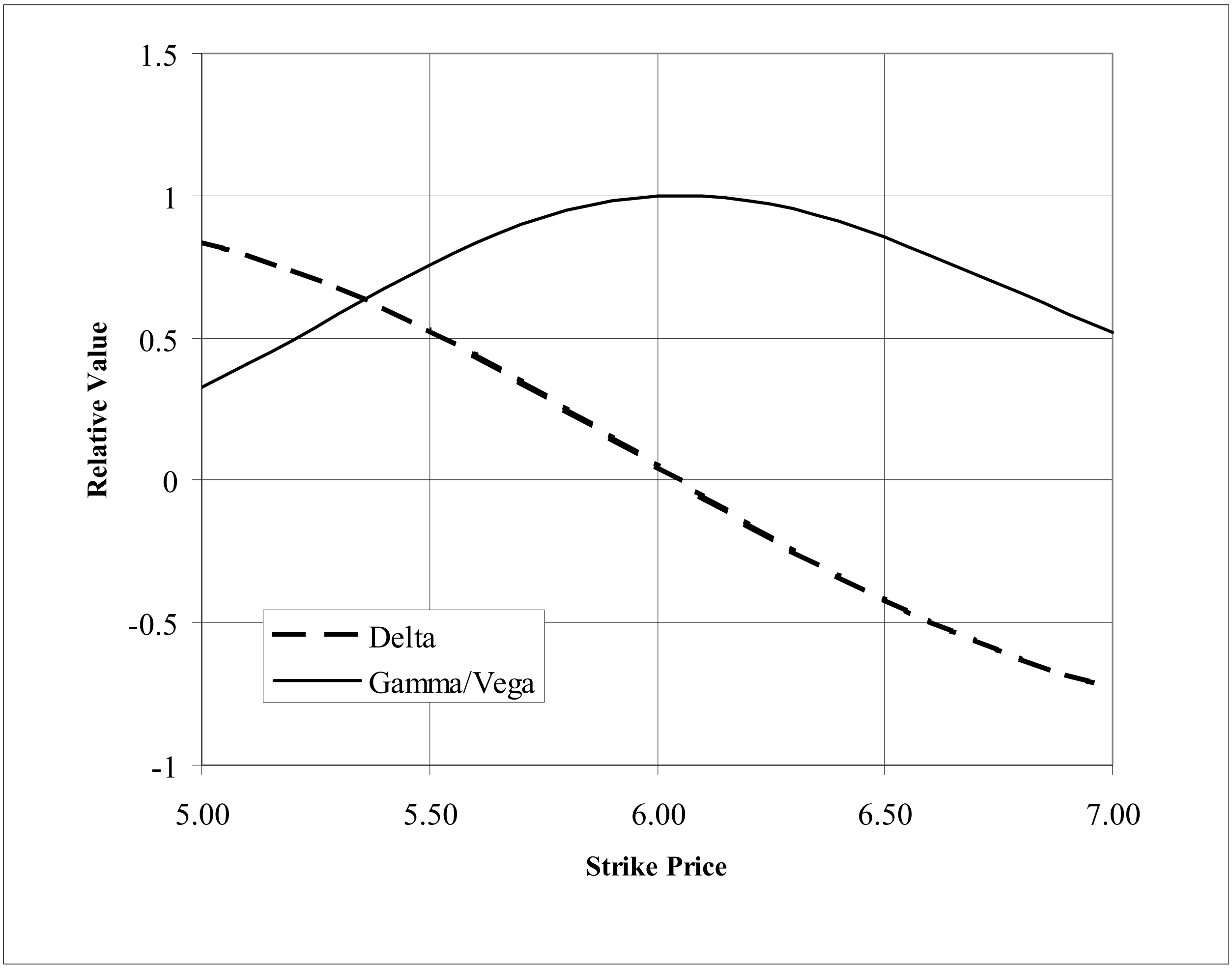

Figure 4: Straddle Greeks as a Function of the Strike Price. Delta, gamma, vega, and theta are simulated at different strike prices for a Eurodollar straddle using the Black model for the case when F=6.00, r=6%, σ =.18, and t=.5 (years). The Greeks are expressed as a percent of their maximum values.

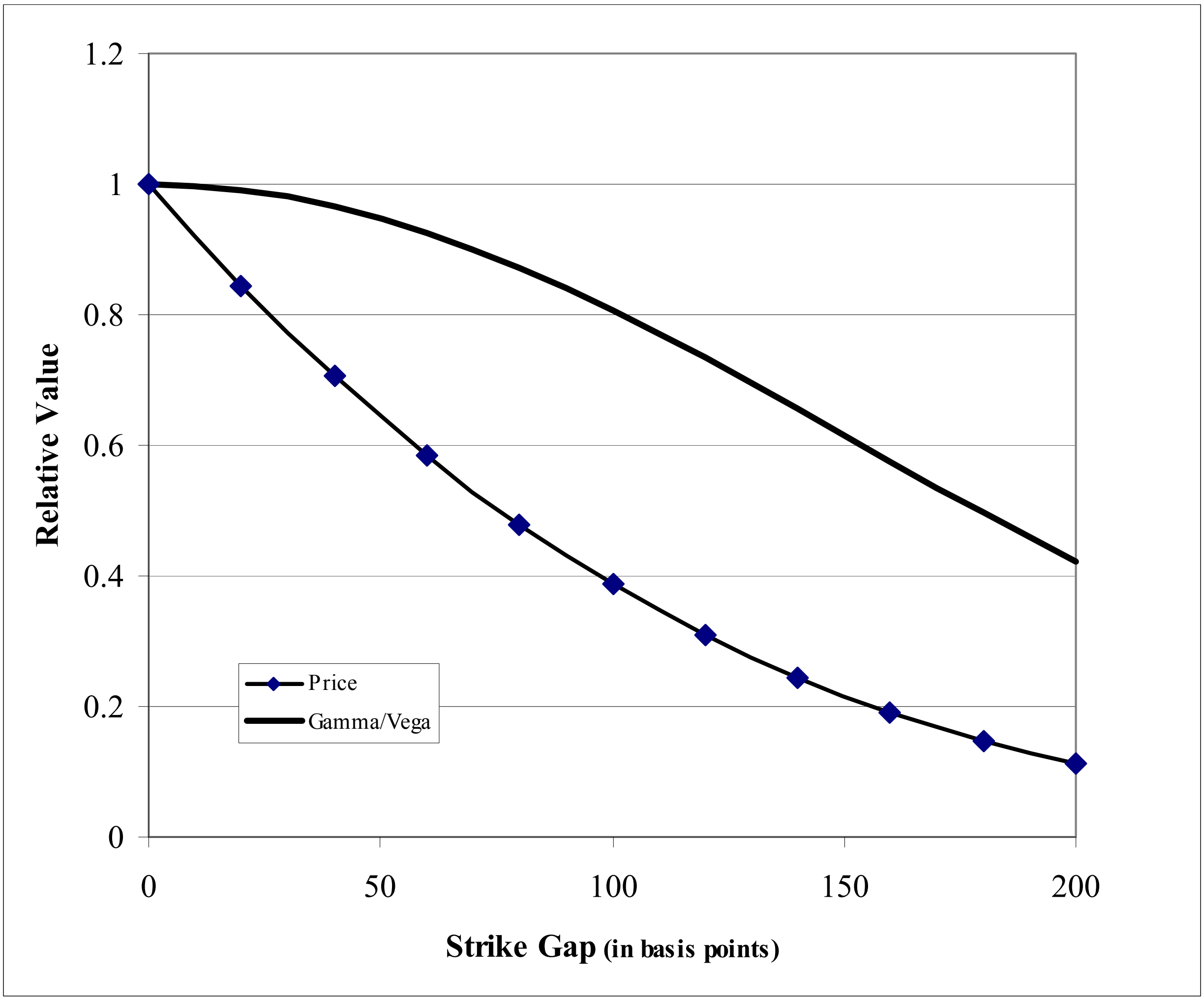

Figure 5: Strangle Greeks as a Function of the Strike Price Differential. Combination characteristics are calculated for a Eurodollar strangle as a function of the gap between the two strikes using the Black model for the case when r=6%, σ=.18, t=.5 (years), F*=6.0, and the mean of the two strikes is 6.0. The parameter values are expressed as a percent of their value when the gap=0 (a straddle).

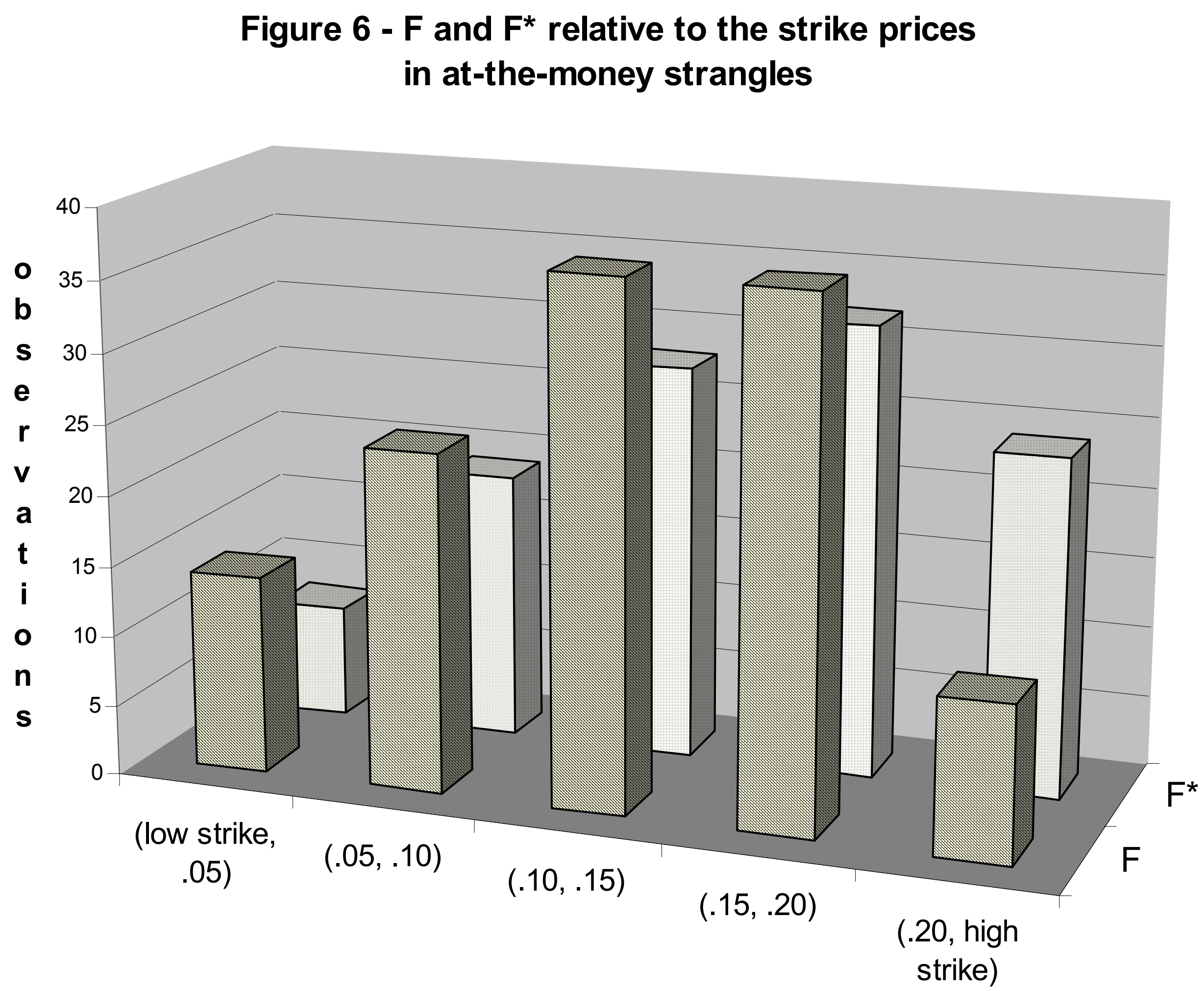

Figure 6. The gap in basis points between the two strike prices in close-to-the-money strangles with a 25 bp (or 12.4 adjusted to 25 bp) differential is divided into five 5 basis point quintiles. The number of times the futures price, F, and the zero delta price, F*, fall in each quintile are shown.

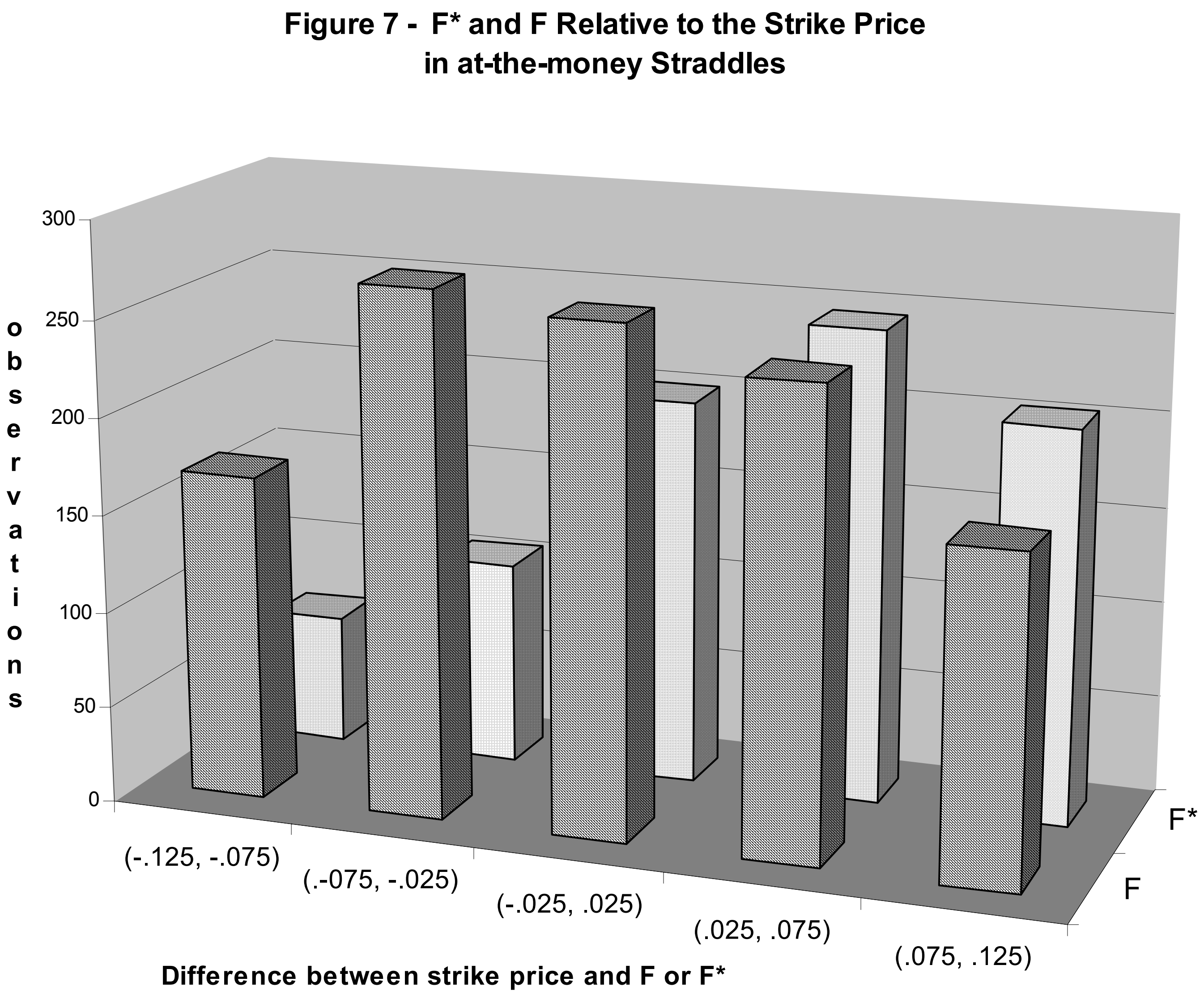

Figure 7 - Statistics on the difference between the strike price and the futures price, F, or between the strike and the zero-delta price, F*, are reported for at-the-money straddles. The 25 (or 12.5 adjusted to 25) basis point region around F or F* is divided into five 5-bp sub-regions and the number of times F or F* falls in each quintile is reported.

Table 1 - Volatility Spread Definitions	
Volatility spreads as defined by the Chicago Mercantile Exchange. All descriptions are for long positions and are expressed as one combination unit.. All options in a spread have the same time-to-expiration.	
Name	**Definition**
Straddle	Buy a call and a put with the same strike price.
Strangle	Buy a put and buy a call at a higher strike price.
Gut	Buy a call and buy a put at a higher strike price.
Butterfly	Buy a call(put), sell two calls (puts) at a higher strike price and buy a call (put) at yet a higher strike price.
Condor	Buy a call(put), sell calls (puts) at two higher strike prices and buy a call (put) at yet a higher strike price.
Iron Butterfly	Buy a straddle and sell a strangle.

Table 2
Black Model "Greeks" for Calls, Puts, Straddles, Strangles, and Butterflies

Derivatives according to Black's options on futures model are presented where: F is the underlying futures price, X the exercise price, P the price of the option, σ the volatility, t is the time-to-expiration, and r is the risk-free interest rate. $d = [\ln(F/X) + .5\sigma^2 t] / \sigma / t$. N(.) represents the cumulative normal distribution, and n(.) the normal density. All derivatives are for positions which are long volatility and are reversed for short positions. For straddles and strangles, the subscripts c and p designate the call and put strikes respectively. For butterflies, the subscripts 1,2, and 3 designate the three different options. In this table calls and puts are defined in terms of LIBOR, not 100-LIBOR. In the butterfly expression for delta, it is assumed the spread is constructed using calls.

	Delta (MP/MF)	Gamma (M^2P/MF^2)	Vega (MP/Mσ)	Theta (MP/Mt)
Call	$e^{-rt}N(d)$	$\frac{e^{\&rt}}{F\sigma\sqrt{t}} n(d)$	$Fe^{\&rt}\sqrt{t}\, n(d)$	$\frac{e^{\&rt}F\sigma}{2\sqrt{t}} n(d) \& rP$
Put	$e^{-rt}[N(d)-1]$	$\frac{e^{\&rt}}{F\sigma\sqrt{t}} n(d)$	$Fe^{\&rt}\sqrt{t}\, n(d)$	$\frac{e^{\&rt}F\sigma}{2\sqrt{t}} n(d) \& rP$
Straddle	$e^{-rt}\,[2N(d)-1]$	$\frac{e^{\&rt}}{F\sigma\sqrt{t}} [2\, n(d)]$	$Fe^{\&rt}\sqrt{t}\, [2\, n(d)]$	$\frac{e^{\&rt}F\sigma}{2\sqrt{t}} [2\, n(d) \& r(P_c \% P_p)]$
Strangle (& Gut)	$e^{-rt}\,[N(d_c)+N(d_p)-1]$	$\frac{e^{\&rt}}{F\sigma\sqrt{t}} [n(d_c) \% n(d_p)]$	$Fe^{\&rt}\sqrt{t}\, [n(d_c) \% n(d_p)]$	$\frac{e^{\&rt}F\sigma}{2\sqrt{t}} [n(d_c) \% n(d_p) \& r(P_c \% P_p)]$
Butterfly (& iron)	$e^{-rt}\,[-N(d_1)+2N(d_2)-N(d_3)]$	$\frac{e^{\&rt}}{F\sigma\sqrt{t}} [\& n(d_1) \% 2\, n(d_2) \& n(d_3)]$	$Fe^{\&rt}\sqrt{t}\, [\& n(d_1) \% 2n(d_2) \& n(d_3)]$	$\frac{e^{\&rt}F\sigma}{2\sqrt{t}} [\& n(d_1) \% 2n(d_2) \& n(d_3) \& r(\& P_1 \% 2P_2 \& P_3)]$

Table 3 - Illustrative Characteristics of Volatility Spreads

Prices, expected payoffs at expiration and "Greeks" of the various volatility spreads are derived using both the Black model (Panel A) and the Barone-Adesi-Whaley (BW) American options model (Panel B) All are constructed for Eurodollar options assuming the underlying LIBOR rate is 6.465%, volatility is 16% (the approximate mean in our sample), and time-to-expiration is 5 months. All six volatility trades are constructed so that the mean strike equals 6.50%, i.e., they are centered at-the-money. For strangles, guts, and butterflies we present estimates for both the case when the strikes differ by 25 basis points and when they differ by 50. For iron butterflies and condors, strikes differ by 25 basis points.

Spread	Strikes	Price (in basis points)	Expected Payout (bp)	Delta	Gamma	Vega	Theta
Panel A - Black Model							
Straddle	6.500	52.040	53.469	0.0000	1.1630	3.2409	0.5884
Strangle	6.375, 6.625	40.770	41.889	-0.0014	1.1430	3.1852	0.5851
	6.250, 6.750	31.265	32.123	-0.0052	1.0850	3.0235	0.5602
Gut	6.375, 6.625	65.102	66.889	-0.0014	1.1430	3.1852	0.5692
	6.250, 6.750	79.929	82.123	-0.0052	1.0850	3.0235	0.5286
Butterfly	6.25, 6.50, 6.75	-3.557	-3.655	0.0052	0.0780	0.2174	0.0441
	6.00, 6.50, 7.00	-13.787	-14.166	0.0169	0.2828	0.7882	0.1603
Iron Fly	6.25, 6.50, 6.75	20.775	21.345	-0.0052	0.0780	0.2174	0.0282
Condor	6.125, 6.375, 6.625, 6.875	-7.002	-7.194	0.0094	0.1484	0.4137	0.0840
Panel B - Barone-Adesi-Whaley Model							
Straddle	6.500	53.040	54.496	0.0003	1.1793	3.2633	0.6036
Strangle	6.375, 6.625	41.046	42.173	-0.0012	1.1555	3.2070	0.5976
	6.250, 6.750	31.476	32.340	-0.0052	1.0946	3.0438	0.5704
Gut	6.375, 6.625	65.571	67.371	-0.0012	1.1555	3.2066	0.5876
	6.250, 6.750	80.540	82.751	-0.0052	1.1128	3.0415	0.5505
Butterfly	6.25, 6.50, 6.75	-3.606	-3.705	0.0063	0.0758	0.2202	0.0432
	6.00, 6.50, 7.00	-13.999	-14.383	0.0216	0.2732	0.8028	0.1567
Iron Fly	6.25, 6.50, 6.75	20.923	21.497	-0.0054	0.0848	0.2195	0.0332
Condor	6.125, 6.375, 6.625, 6.875	-7.103	-7.298	0.0116	0.1438	0.4201	0.0822

Table 4 - The Implied Volatility Smile for Eurodollar Options							
Implied volatility means are reported based on daily settlement prices for the periods 5/10/94-5/18/95 and 4/18/99-7/31/00. In the "Strike Price" column, the first letter (C or P) stands for a **C**all or a **P**ut; the second letter (I or O) refers to **I**n-the-money or **O**ut-of-the-money; and the last digit indicates the relative position of an option from the money where 1 indicates that the option is the nearest-to-the-money and 2 indicates that the option is the second nearest-to-the-money etc. We also report means for a measure (X/F -1) of how far in or out of the money the strike is where X= strike price and F=underlying Eurodollar futures price.							
Calls				**Puts**			
Strike Price	**Mean Implied Standard Deviation**	**Mean K/F -1**	**Obs**	**Strike Price**	**Mean Implied Standard Deviation**	**Mean K/F -1**	**Obs**
Panel A - Options Maturing in 2 to 6 Weeks							
CI4	15.62%	-0.0770	132	PO4	14.80%	-0.0770	221
CI3	13.64%	-0.0498	128	PO3	13.60%	-0.0502	179
CI2	12.82%	-0.0304	168	PO2	12.34%	-0.0309	197
CI1	11.93%	-0.0107	206	PO1	11.96%	-0.0107	206
CO1	12.60%	0.0097	197	PI1	12.74%	0.0097	192
CO2	13.15%	0.0290	204	PI2	14.09%	0.0209	156
CO3	14.35%	0.0496	183	PI3	15.73%	0.0502	124
CO4	16.28%	0.0788	314	PI4	16.40%	0.0772	163
Panel B - Options Maturing in 13 to 26 Weeks							
CI8				PO8	16.20%	-0.2703	114
CI7	16.28%	-0.2366	76	PO7	16.48%	-0.2321	126
CI6	16.13%	-0.1958	93	PO6	17.23%	-0.1988	161
CI5	15.47%	-0.1607	140	PO5	16.89%	-0.1689	233
CI4	14.34%	-0.1316	239	PO4	15.64%	-0.1343	331
CI3	13.67%	-0.0961	335	PO3	14.54%	-0.0978	377
CI2	13.74%	-0.0593	377	PO2	14.11%	-0.0593	377
CI1	14.31%	-0.0199	389	PO1	14.31%	-0.0199	391
CO1	14.74%	0.0195	379	PI1	14.75%	0.0194	376
CO2	15.25%	0.0583	379	PI2	15.31%	0.0583	359
CO3	15.84%	0.0972	377	PI3	16.12%	0.0975	293
CO4	16.59%	0.1357	363	PI4	16.17%	0.1362	212
CO5	17.45%	0.1743	349	PI5	17.16%	0.1746	157
CO6	18.29%	0.2125	297	PI6	17.57%	0.2121	77
CO7	19.00%	0.2506	215	PI7			
CO8	19.53%	0.2908	116	PI8			

Table 5 - Volatility Spread Trading

Figures are based on all option trades of 100 contracts or more in the Eurodollar options market on 385 trading days during the 1994-1995 and 1999-2000 periods. For each of the six volatility spreads listed we report statistics on their trading as a percent of (1) all trades of 100 contracts or more, (2) all spread and combination trades, and (3) the six volatility spreads.

Panel A - Number of Trades

Spread	Number of Trades	Percent of all large trades	Percent of spreads and combinations	Percent of volatility trades
Straddles	2379	17.50%	30.32%	73.07%
Strangles	676	4.97%	8.61%	20.76%
Guts	10	0.07%	0.13%	0.31%
Butterflies	154	1.13%	1.96%	4.73%
Iron Butterflies	28	0.02%	0.36%	0.86%
Condors	9	0.07%	0.11%	0.28%

Panel B - Volume

Spread	Total Volume in Contracts (000's)	Percent of all large trades	Percent of spreads and combinations	Percent of volatility trades
Straddles	2658.1	13.76%	18.56%	58.94%
Strangles	1263.2	6.54%	8.82%	28.01%
Guts	17.2	0.09%	0.12%	0.38%
Butterflies	477.1	2.47%	3.33%	10.58%
Iron Butterflies	66.5	0.34%	0.46%	1.48%
Condors	27.8	0.14%	0.19%	0.62%

Table 6 - Descriptive Statistics

Means and medians are reported for various characteristics of straddle, strangle, and butterfly trades based on option trades of 100 contracts or more in the Eurodollar options market on 385 trading days during the 1994-1995 and 1999-2000 periods. For the Greek and implied volatility calculations, the underlying futures price at the time of the trade is approximated using an average price for that day. Accurate implied volatilities for the butterfly trades could not be obtained due to the low vegas.

Characteristic	**Straddles**		**Strangles**		**Butterflies**	
	Mean	**Median**	**Mean**	**Median**	**Mean**	**Median**
Panel A - General Characteristics						
Price (in bp)	63.53	60.00	26.06	22.00	6.51	5.00
Expiry (months)	7.94	6.59	5.03	4.34	3.55	2.96
Size (contracts)	1101	1000	2018	1000	3015	2000
Implied Volatility	16.14%	16.66%	15.65%	15.65%	NA	NA
Panel B - Mean Absolute Greeks According to the Black Model						
Delta	0.156	0.109	0.135	0.104	0.074	0.062
Gamma	1.565	0.991	1.393	1.151	0.222	0.098
Vega	3.596	3.533	2.466	2.434	0.246	0.168
Theta	0.530	0.456	0.527	0.47	0.096	0.055
n(d)’	0.766	0.790	0.629	0.699	0.086	0.057
Panel C - Mean Absolute Greeks According to the Barone-Adesi-Whaley Model						
Delta	0.156	0.102	0.135	0.104	0.074	0.062
Gamma	1.579	1.004	1.398	1.158	0.220	0.098
Vega	3.655	3.576	2.485	2.444	0.247	0.168
Theta	0.572	0.514	0.543	0.491	0.095	0.054
Observations	1751		530		91	

Table 7 - Straddle Strike Choices and Implications

We report on which strike prices are chosen for straddles and how this choice impacts the straddle's delta according to the Black and Barone-Adesi-Whaley models and gamma and vega according to the Black model. X is the chose strike price; X* is the strike (among those traded) at which delta is minimized and gamma and vega maximized according to the Black model; and X_m is the available strike which is closest to the underlying asset price (which may be the same as X*). For a given expiry gamma and vega are proportional to n(d)ℕ, the value of the normal density function at strike price X.

Strike Chosen	**Number**	**Percent**	**Mean Absolute Black Delta**	**Mean Abs. Black Delta if X=X***	**Mean Absolute BW Delta**	**Mean Abs. BW Delta if X=X***	**Mean n(d)**	**Mean $n(d)_c$ if X=X***
X=X*=X_m	880	50.3%	.0951	.0951	.0957	.0957	.7501	.7501
X=X* but X…X_m	71	4.0%	.0515	.0515	.0533	.0533	.7637	.7637
X=X_m but X…X*	364	20.8%	.1150	.0517	.1177	.0542	.7134	.7667
Next closest strike to X* or X_m	366	20.9%	.2993	.0971	.2988	.0979	.6703	.7589
More than one strike from X* or X_m	70	4.0%	.5030	.0841	.5013	.0851	.5382	.7626
All	1751	100.0%	.1564	.0843	.1585	.0854	.7179	.7565

Table 8
Strangle (Straddle) Characteristics by Strike Price Differential

Mean values of several strangle (and straddle) characteristics are reported for different gaps or differentials between the strike prices of the call and put. p-values are also reported for ANOVA tests of the null hypothesis that the means do not differ by strike price differential.

Combination Characteristic	Strike Price Differential (D) (in basis points)					Test of equality null (p-value)	
	D=0 Straddles	D#25	25<D#50	50<D#100	100<D	strangles only	with straddles
Mean Differential (D) (in basis points)	0	24.0	50.0	87.8	163.1	.001	.001
Time-to-Expiration (months)	7.94	3.37	5.01	6.89	6.79	.001	.001
Net Price (basis points)	63.53	24.71	27.93	29.60	19.19	.002	.001
Absolute Black Delta	.156	.151	.130	.135	.099	.024	.004
$n(d)_c$	.766	.715	.679	.601	.456	.001	.001
Gamma	1.565	2.144	1.248	.841	.530	.001	.001
Vega	3.596	2.164	2.622	2.823	2.328	.001	.001
Theta	.530	.654	.537	.403	.335	.001	.001
Percent Short Volatility Positions	53.9%	66.7%	64.2%	63.2%	45.6%	.091	.001
Observations	1751	186	173	106	65		

Table 9 - Probit Analysis of the Straddle-Strangle Choice

Results are presented for a probit analysis where the choice variable is coded as 0 for a straddle and 1 for a strangle. TTE represents the time-to-expiration in years of the options. L/S equals 0 if the position is long volatility and 1 if short. Z=|F-closest strike| where F is the underlying futures price. Z*=|F*-closest strike| where $F^* = F e^{.5\sigma^2 t}$. TTE*Z is the product or Z and TTE. SMILE = V_2/V_1 if the straddle/strangle position is long and = $-(V_2/V_1)$ if the straddle/strangle position is short where V_1 is the average implied volatility at strikes within 20 basis points of F and V_2 is the average implied volatility at traded strikes more than 20 basis points from F. The sample consists of all strangles where the gap between the two strikes is 25 basis points and either F or F* is between the two strikes, and all straddles at <u>either</u> of the <u>two</u> strikes closest to F or F*. Z statistics are shown in parentheses below the coefficients. Significance at the .10, .05, and .01 levels is indicated by *, **, and *** respectively.

Independent Variable	Model 1	Model 2	Model 3
Intercept	-1.117*** (-7.26)	-1.357*** (-6.73)	-1.543*** (-3.02)
TTE	-1.749*** (-7.45)	-0.906* (-1.82)	-1.771*** (-7.50)
L/S	0.243* (1.90)	0.232* (1.82)	1.102 (1.11)
Z	8.171*** (4.60)	13.363*** (3.97)	8.206*** (4.62)
Z*	-1.223 (-0.76)	-3.088 (-1.57)	-1.168 (-0.73)
TTE*Z		-10.317* (-1.77)	
SMILE			0.387 (0.88)

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ENDNOTES

1. For example of textbook discussions see Hull (2000) or Kolb (2000). For a particularly good treatment in the practitioner literature, see Natenberg (1994).

2. By convention, the first derivatives of an option's price to price, volatility, time, and the interest rate are known as "delta", "vega", "theta", and "rho" respectively while the second derivative with respect to the underlying asset price is termed "gamma."

3. We ignore rho since it is negligible for all but very long term options. We discuss theta but note that it is not a risk measure since time to expiration is known with certainly.

4. Based on conversations with traders and personal observations by one author who worked in the Eurodollar options pit Black's model is by far the most popular. As we shall see below, there is also evidence from our data that this is the model being employed by most volatility traders in this market.

5. Longer term guts would be an exception to this statement.

6. As explained more fully in Chaput and Ederington (2002), option terminology in the Eurodollar market can be confusing. Although Eurodollar futures and options are officially quoted in terms of 100-LIBOR, in calculating option values in the Eurodollar market, traders generally use pricing models, such as the Black model, defined in terms of LIBOR, not 100-LIBOR. For instance, consider a Eurodollar call with an exercise price of 94.00. This call will be exercised if the futures price (100-LIBOR) is greater than 94, or if LIBOR<6.00%. So a call in terms of 100-LIBOR is equivalent to a LIBOR put and vice versa. Hence, the price of a Eurodollar <u>call</u> as officially quoted is obtained by setting F=LIBOR (not 100-LIBOR), X=6.00 (not 94.00), and σ defined in terms of LIBOR rate volatility into the pricing equation for a <u>put</u>. Indeed, this is the procedure used by the exchange to obtain its official volatility quotes. This is the procedure used in Table 3 and throughout this paper.

7. While it has been hypothesized that spreads are lower on combination orders than they would be on the individual options making up the combination, Chaput and Ederington (2002) find no evidence to support this hypothesis.

8. However, one broker has told us that he may negotiate away one of these charges for his best customers.

9. Eurodollar futures contracts are cash-settled contracts on the future 3-month LIBOR rate. Since LIBOR is a frequent benchmark rate for variable rate loans, and interest rate swaps, hedging opportunities abound so this is a very active market.

10. The 100 contract floor above which trades are recorded refers to each leg. For instance, if an order is received for 80 straddles (80 calls and 80 puts), it is not recorded even though a total of 160 options are traded while an order for 100 naked calls would be.

11. Since the time of the trade is not recorded, we do not know the exact price of the underlying Eurodollar futures at the time of the trade unless the order includes a simultaneous futures transaction, e.g., a covered call.

12. The Futures Industry Institute data does not report implied volatilities for the April 1999 - September 1999 period so these figures are based on 1994-1995 and 2000.

13. For options maturing in three months or more, Eurodollar options are traded in increments of 25 basis points. For options maturing in less than three months, the traded

strikes after May 1995 are 12.5.basis points apart.

14. Constant maturity 3-month T-bill rates are used for options expiring in less than 4.5 months, 6-month T-Bills for options maturing in 4.5 to 7.5 months, 9-month for options expiring in 7.5 to 10.5 months and 1-year rates for all longer options.

15. As shown in Table 2, the expression for theta is more complicated but, for except for very long term or deep in-the-money options, the term rP tends to be small so theta's maximum also occurs close to F*. Unfortunately, this means that the holder of a long straddle where X=F* loses considerably if his volatility bet is incorrect. Likewise, the holder of a short straddle maximizes her theta (approximately) by choosing X=F* but also maximizes her vega and gamma risks.

16. Prior to May 1995 all were in 25 basis point increments.

17. The lost observations are because one of the ISDs could not be calculated because the relevant option did not trade that day.

18. The true percentage could be higher because we only observe the futures trades which were part of the same order. If a trader placed one order for the straddle and a separate order for the futures, it would be counted as a straddle without futures in our data set.

19. Since the available strikes are in increments of 12.5 o5 25 basis points this cannot be done precisely in practice. For example if the strikes are 6.00 and 6.25, changing them to 5.75 and 6.50 leaves the arithmetic average unchanged at 6.125 but changes the geometric average from 6.124 to 6.114.

20. Also, for the same strikes, as time to expiration declines, d_c and d_p become larger in absolute terms so $[n(d)_c + n(d)_p]$ declines which tends to reduce gamma and vega. Since gamma is proportional to $1/\sqrt{t}$, it still normally tends to increase but vega is proportional to $\sqrt{t}$ so declines sharply.

21. By basing V_2 on all traded away-from-the-money strikes we compensate for the flatter smile slope at the longer maturities. At the longer expiries, the smile tends to be flatter but strikes further away from the money are traded that day.